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NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

Local Geoid Determination
Using the Global Positioning System

by

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September 1988

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Unclassified

Security Classification of this page

REPORT DOCUMENTATION PAGE

1a Report Security Classification Unclassified	1b Restrictive Markings
2a Security Classification Authority	3 Distribution Availability of Report
2b Declassification/Downgrading Schedule	Approved for public release; distribution is unlimited.
4 Performing Organization Report Number(s)	5 Monitoring Organization Report Number(s)
6a Name of Performing Organization Naval Postgraduate School; 6b Office Symbol (If Applicable) code 68	7a Name of Monitoring Organization Naval Postgraduate School
6c Address (city, state, and ZIP code) Monterey, CA 93943-5000	7b Address (city, state, and ZIP code) Monterey, CA 93943-5000
8a Name of Funding/Sponsoring Organization; 8b Office Symbol (If Applicable)	9 Procurement Instrument Identification Number
8c Address (city, state, and ZIP code)	10 Source of Funding Numbers: Program Element Number; Project No; Task No; Work Unit Accession No

11 Title (Include Security Classification) Local Geoid Determination using the Global Positioning System

12 Personal Author(s) Ma, Wei-Ming

13a Type of Report	13b Time Covered	14 Date of Report (year, month,day)	15 Page Count
Master's Thesis	From To	1988 September	94

16 Supplementary Notation The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

17 Cosati Codes			18 Subject Terms (continue on reverse if necessary and identify by block number)
Field	Group	Subgroup	Global Positioning System, GPS, geoid, geoid height, ellipsoid height; orthometric height; double difference fixed solution; the method of collocation.

19 Abstract (continue on reverse if necessary and identify by block number

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The data used in this research were taken from GPS measurements on the campus of the Naval Postgraduate School (NPS), an area about 100 m x 630 m, and in an area approximately 15 km x 33 km near Monterey, California. The time period was from February 5, 1988, to May 12, 1988.

The accuracy of the predicted geoid heights is ±2 cm if a six-parameter model is used for the larger area, and ± 2 to 10 mm if a five-parameter model is used for the NPS campus.

20 Distribution/Availability of Abstract [X] unclassified/unlimited [ ] same as report [ ] DTIC users	21 Abstract Security Classification Unclassified
22a Name of Responsible Individual Stevens P. Tucker	22b Telephone (Include Area code) (408) 646-3269; 22c Office Symbol code 68Tx

DD FORM 1473, 84 MAR 83 APR edition may be used until exhausted security classification of this page

All other editions are obsolete Unclassified

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Local Geoid Determination

Using the Global Positioning System

by

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Submitted in partial fulfillment of the requirements
for the degree of

MASTER OF SCIENCE IN HYDROGRAPHIC SCIENCES

from the

NAVAL POSTGRADUATE SCHOOL
September 1988

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ABSTRACT

A local geoid model to predict the geoid heights in the vicinity of Monterey Bay, California, was developed to use Global Positioning System (GPS) differential positions and known Mean Sea Level (MSL) with the method of collocation. The local geoid models were based on Rapp's 360 degree x 360 order global geoid model determined from gravity measurements. Control data were adjusted by least squares to solve for the parameters in the local geoid model. Also studied were factors that affected the GPS-measured ellipsoid height differences. These included (1) comparing GPS differencing solutions, (2) standard error of GPS observations, (3) corrections for surface meteorological values, and (4) observation durations for GPS.

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ACKNOWLEDGMENTS

I would like to express my sincere appreciation to Professors Kandiah Jeyapalan and Stevens P. Tucker, my thesis advisors, for their dedicated assistance and guidance during the study. Without their encouragement, patience and enthusiasm this project could not have come to a successful completion.

I would also like to thank Cdr. Kurt J. Schnebele, NOAA, and Geodesist James R. Cherry for their technical instruction and assistance.

I would like to thank Lt. Richard B. Koehler, NOAA, for assisting in collecting GPS data in the field at night and for helping me to learn to use the Macintosh Plus computer.

I would like to thank Capt. Mohammad A. Khalid, Pakistan Navy, Mr. Kenneth E. Welker and Lt. Lung, Ming-Hung, Republic of China Navy, for their help performing levelling on the NPS campus.

Also, I would like to thank Lt. Derek D. Hee, USN, for assisting me in collecting GPS data and for proofreading my thesis.

Finally, I would like to thank my wife Nim-Chi and son Hsuan-Ho for their love and encouragement.

I. INTRODUCTION

The Global Positioning System (GPS) is able to establish precise relative positions in the World Geodetic System of 1984 (WGS 84). A Trimble GPS receiver, which has the capability of measuring carrier phase, was used to determine the vector base line in space. The components of the base line are expressed in terms of cartesian coordinate differences (ΔX, ΔY, ΔZ) [Remondi, 1984]. These vector base lines can be converted to distances, azimuths and the ellipsoid height differences, Δh, relative to the WGS 84 Ellipsoid.

The results of several tests and operations have clearly shown that GPS survey methods can replace conventional horizontal survey methods. Comparable accuracies have also been achieved for GPS-derived ellipsoid height differences, Δh. The problem of converting these ellipsoid height differences, Δh, to orthometric height differences, ΔH, remains to be resolved. For example, in engineering surveying applications WGS 84 coordinates must be transformed to the North American Datum of 1983 (NAD 83) system. The GPS obtains ellipsoid heights, rather than the orthometric heights; the geoid height, N, must be calculated to obtain the latter. One of the problems in this transformation is the accurate determination of the local geoid height, N.

For GPS survey applications, a geoid model should provide geoid heights with an accuracy commensurate with that of the ellipsoid height, h, so the accuracy of the derived orthometric heights is not reduced. In the future differential positioning will be widely used in GPS surveying, and, therefore, only the geoid height differences between stations will be required.

Geoid height computation techniques include the following:

(1) Geoid height differences in the U.S. can be determined from gravity data and the Stokes' integral method, or from astrogravimetric data and least squares collocation methods. These methods lead to uncertainties that are typically 1 to 10 cm for distances of less than 20 km and 5 to 20 cm for distances between 20 to 50 km [Zilkoski, 1988]

(2) A comparison of a data set from GPS with gravimetrically determined geoid heights using least squares collocation techniques shows discrepancies between the two data sets of about ± 2 cm for a maximum intersection distance of approximately 50 km [Denker and Wenzel, 1987].

(3) Mean gravity anomalies, deflections of the vertical and a geopotential model calculated to degree and order 180 have been used to determine geoid heights in the area bounded by ($34° \leq \varphi \leq 42°$, $18° \leq \lambda \leq 28°$) [Tziavos, 1987]. The method used was that of least squares collocation. By using empirical covariance functions for the data, suitable weighting functions for the different sources of observations, and the optimum cap radius around each point of elevation, an accuracy better than ± 0.60 m was obtained for geoid heights.

The main objective of this thesis is to use a model that predicts N in a region near Monterey, California, from the GPS differential positions and known mean sea level (MSL) using the method of collocation. Also studied were factors that affected the ellipsoid height differences, Δh, obtained from GPS measurements, such as comparing GPS differencing solutions, standard error of GPS observations, corrections for surface meteorological value , and observation durations for GPS measurements. The data used in this research were taken from GPS measurements on the campus of the Naval Postgraduate School (NPS) an area about 100 m x 630 m and in an area approximately 15 km x 33 km near Monterey, California. The time period was from February 5, 1988, to May 12, 1988.

The accuracy of the predicted geoid height is ± 2 cm if a six-parameter model is used for the large area, and ± 2 to 10 mm if a five-parameter model is used for the NPS campus.

II. GEOID HEIGHT

A. THE GEOID

Surveyors and engineers, in most cases, are interested in the orthometric height, H, as measured above the reference surface of the geoid. The ocean is considered to be freely moving, homogeneous and only subject to the force of gravity. When a state of equilibrium is achieved, the surface of this idealized ocean assumes a level surface of the gravity field. It may be regarded as also extending under the continents. This level surface is called the geoid. If the potential, W, is given as a function of the coordinates r, ϕ and λ, then the geoid is given by [Moritz, 1984]

$$W = W(r,\phi,\lambda) = W_0$$

The geoid is a closed and continuous level surface which extends partially inside the solid body of the earth. The direction of the gravity vector at any point (plumb line or vertical) is normal to the geoid. The curvature of the geoid displays discontinuities at abrupt density variations. Consequently, the geoid is not an analytic surface, and therefore not a practical reference surface for position determinations. The geoid however, is well suited as a reference surface for potential or height differences, which are obtained by precise levelling in combination with gravity measurements.

To establish the geoid as a reference surface for heights, the ocean water level is recorded and averaged over long intervals ($\geq$ 1 year) using tide gauges. The MSL thus obtained represents an approximation to the geoid. The National Geodetic Vertical Datum of 1929 (NGVD 29) was derived for land surveys from a general adjustment of the first order levelling net of both United States of America and Canada. In the adjustment MSL was observed at twenty-one tide stations in the United States and five in Canada. The geoid established by this method may deviate by ± 1 to ± 2 m from a level surface due to periodic, nonperiodic and secular variations [Torge, 1980].

B. THE WGS 84 ELLIPSOID

The development of WGS 84 [DMA, 1987] was initiated by the United States Department of Defense for navigation and weapon systems. The Defense Mapping Agency (DMA) developed WGS 84 as a replacement for WGS 72. The defining parameters and reference frame orientation of the WGS 84 Ellipsoid and the WGS 84 Ellipsoid Gravity Formula are those of the internationally sanctioned Geodetic Reference System of 1980 (GRS 80). Accordingly, a geocentric equipotential ellipsoid is defined by the semimajor axis (a), the flattening (f), the equatorial gravity (γ_a), and the angular velocity (ω). The WGS 84 Ellipsoid used the values:

$$
\begin{aligned}
a &= 6378137 \text{ m} \\
f &= 1/298.257223563 \\
\gamma_a &= 987.03267714 \text{ gals} \\
\omega &= 7.292115\text{x}10^{-5} \text{ rad/s}
\end{aligned}
$$

The reference system for GPS is WGS 84. The precise geocentric coordinates obtained from GPS receivers are in WGS 84.

C. THE ORTHOMETRIC HEIGHT, H, THE ELLIPSOID HEIGHT, h AND THE GEOID HEIGHT, N

In geodetic applications three different surfaces or earth figures are normally involved. First is the earth's actual topography; second, the geometric surface, or ellipsoid and; third, the equipotential surface, the geoid. The relationship between the earth's topography, the ellipsoid and the geoid in a section through the earth's surface is shown in Figure 1.

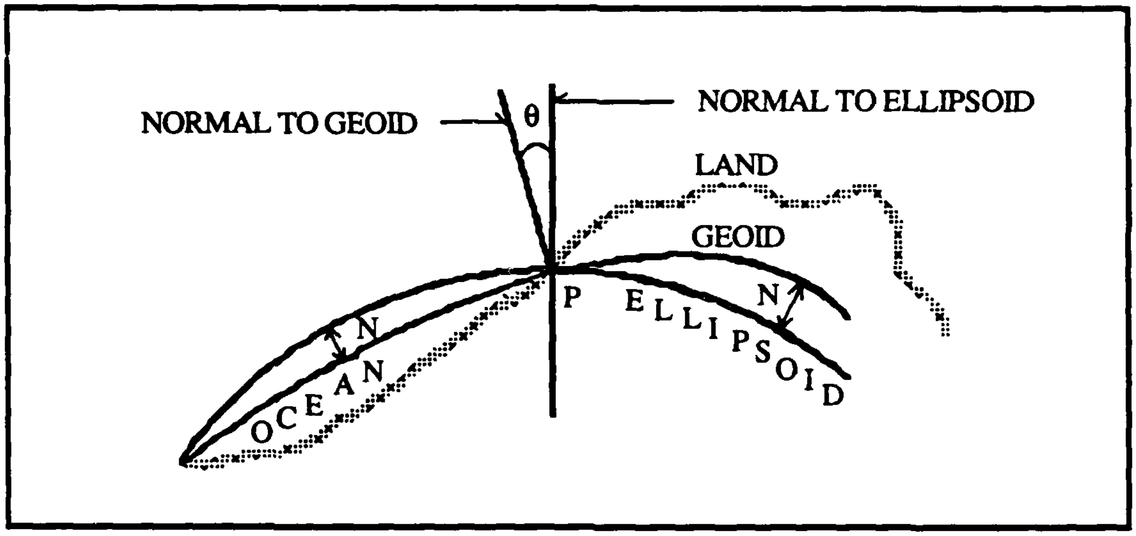

Figure 1. Relationship between the earth's surface, the geoid and the ellipsoid.

Two features in the figure are of particular interest :

1. The deflection of the vertical, θ, defined by Pizzetti as the angle at the geoid between the direction of the plumb line (normal to geoid) and the normal to the ellipsoid through the point, P, on the geoid [Torge, 1980].
2. The vertical separation, N, between the geoid and the ellipsoid.

The deflections of the vertical and the geoid heights, N, depend on the ellipsoidal coordinates and, hence, on the parameters of the reference ellipsoid and its position with respect to the earth. If they are referred to the geocentrically situated mean earth ellipsoid, then they are referred to as absolute quantities; otherwise, they are relative quantities. The absolute deflections of the vertical in flat terrain and the highlands assume values between one and ten seconds of arc; in mountainous areas, they vary between one-half and one minute of arc. As a result of density variations within the earth, the geopotential surfaces, including the geoid, have irregular shapes. Absolute geoid heights, however, rarely exceed 100 m [Torge, 1980].

The orthometric heights, H, shown in Figure 2 are referred to an equipotential surface, the geoid. The orthometric height of a point on the earth's surface is the distance from the reference surface to the point, measured along the plumb line normal to the geoid. The ellipsoid height, h, of a point is the

distance from the reference ellipsoid to the point, measured along the line which is normal to the ellipsoid. For purposes of simplicity, the orthometric height, H, and the ellipsoid height, h, are shown to be along a common vertical. In most cases, this would cause a very small error that is considered insignificant compared to present uncertainties of the geoid height N estimates. The geoid height, N, is defined:

$$N = h - H$$

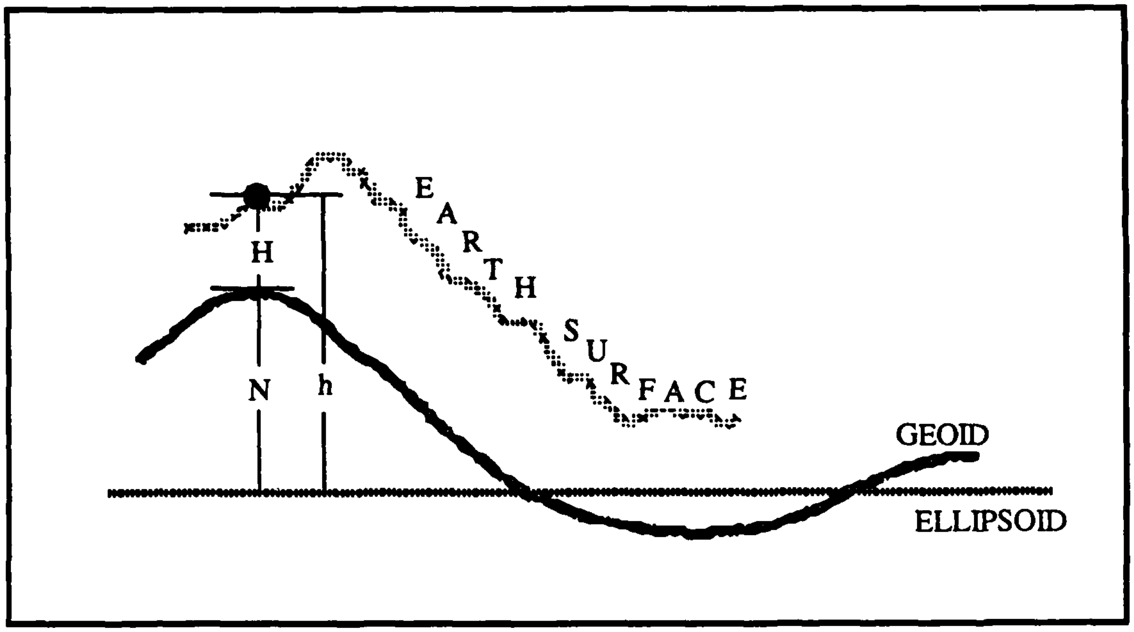

Figure 2. Relationship between the geoid height, N, the ellipsoid height, h and the orthometric height, H.

The orthometric height has greater physical meaning than the geometrical ellipsoid height. The orthometric height has traditionally been determined by the technique of levelling in which increments of height are obtained from the intersection of the line of sight of a level instrument, tangential to the geopotential surface and passing through the level axis, with two graduated rods. Accurate orthometric height information is needed for precise engineering operations such as the construction of dams, pipelines, and tunnels.

Geoid heights have been accurately determined for some major geodetic datums such as the North American, European and Australian. These datums are

well supplied with astrogeodetic deflections and have fair gravity coverage. The standard error of relative geoid heights in these areas is about 2 or 3 m.

D. THE GEOID HEIGHT FROM GRAVITY MEASUREMENT

Both the U.S. National Geodetic Survey (NGS) of National Oceanic and Atmospheric Administration Service (NOAA) and Trimble Navigation versions of Rapp's 360 degree x 360 order model (OSU86F) were available to me. Rapp computed two potential coefficient fields that are complete to degree and order 360 [Rapp, 1986]. One field (OSU86E) excludes geophysically predicted anomalies, while Rapp's other model (OSU86F) includes such anomalies. These fields were computed using a set of 30-minute mean gravity anomalies derived from satellite altimetry in the ocean areas and on land from standard measurements.

Gravity anomalies can be observed and then used to compute the geometric deviation of the geoid from the ellipsoid. The expression for the gravitational potential is written in the following form [Rapp, 1986]:

$$V(r,\phi,\lambda) = \frac{kM}{\gamma}\left[1 + \sum_{l=2}^{\infty}\left[\frac{a}{r}\right]^{l} \sum_{m=0}^{l}\left(\bar{C}_{lm}\cos m\lambda + \bar{S}_{lm}\sin m\lambda \right) \bar{P}_{lm}(\sin\phi)\right]$$

where

r, ϕ, λ : geocentric coordinate

kM : geocentric gravitational constant

a : equatorial radius of the reference ellipsoid

$\bar{C}_{lm}, \bar{S}_{lm}$: fully normalized potential coefficients

$\bar{P}_{lm}$: fully normalized Legendre function of degree l and m

γ : normal gravity

The potential at a point, U, is the scalar sum of V and centrifugal force potential V' [Ewing, 1976]:

$$U(r,\phi,\lambda) = V(r,\phi,\lambda) + V'(r,\phi,\lambda)$$

The difference between observed gravity potential W and the computed normal gravity potential U is denoted by T [Moritz, 1984], so that

$$W(r,\phi,\lambda) = U(r,\phi,\lambda) + T(r,\phi,\lambda)$$

compared the geoid defined by the potential W_0 is given by

$$W(r,\phi,\lambda) = W_0$$

A reference ellipsoid with the same potential, $W_0 = U_0$, is given by

$$U(r,\phi,\lambda) = W_0$$

A point P on the geoid is projected onto the point Q on the ellipsoid along the normal to the ellipsoid PQ = N. PQ is the distance between geoid and ellipsoid at the point and is called the geoid height (Figure 3).

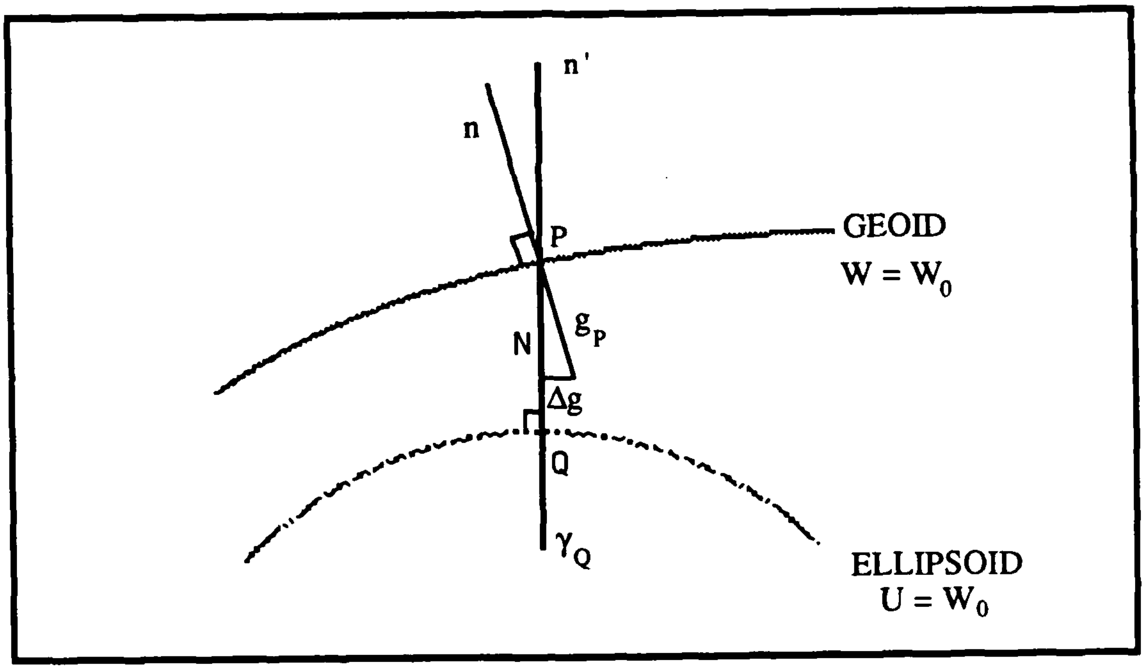

Figure 3. Geoid and ellipsoid (Moritz [1984, Fig. 2-12]).

The disturbing potential T can be written as [Rapp, 1986]:

$$T(r,\phi,\lambda) = \frac{kM}{\gamma} \sum_{l=2}^{\infty} \left[\frac{a}{r}\right]^{l} \sum_{m=0}^{l} \sum_{\alpha=0}^{1} \bar{C}_{lm}^{\alpha} \bar{Y}_{lm}^{\alpha}(\phi,\lambda)$$

where

$$\bar{C}^{\alpha}_{lm} = \{\bar{C}_{lm},\ \alpha = 0,\ \text{and}\ \bar{S}_{lm},\ \alpha = 1\}$$

$$\bar{Y}^{\alpha}_{lm}(\phi,\lambda) = \{\bar{P}_{lm}(\sin\phi)\cos m\lambda,\ \alpha = 0,\ \text{and}\ \bar{P}_{lm}(\sin\phi)\sin m\lambda,\ \alpha = 1\}$$

Since the N is relatively small compared to the ellipsoid geocentric radius R', then [Ewing, 1976]

$$T \doteq \gamma N$$

and

$$\therefore\ N = \frac{T}{\gamma}$$

where γ is the normal gravity at (ϕ, λ).

The gravity anomaly, Δg, in the Molodensky surface free-air anomaly sense is given by [Moritz, 1984]:

$$\Delta g_P = g_P - \gamma_Q$$

The boundary condition that relates Δg and T is [Moritz, 1984]

$$\Delta g = -\frac{\partial T}{\partial h} + \frac{1}{r}\frac{\partial T}{\partial h} T$$

where h is the distance along the plumb line direction. Neglecting deflections of the vertical we have [Rapp, 1986]

$$\Delta g = \frac{kM}{\gamma^2} \sum_{l=2}^{\infty} (l - 1) \left[\frac{a}{r}\right]^l \sum_{m=0}^{\infty} \sum_{\alpha=0}^{1} (\bar{C}^{\alpha}_{lm} - \bar{C}^{\alpha h}_{lm} - \bar{C}^{\alpha h}_{lm} - \bar{C}^{\alpha\gamma}_{lm})\, \bar{Y}^{\alpha}_{lm}(\phi,\lambda)$$

where $\bar{C}^{\alpha i}_{lm}$ ($i = h, \gamma$) are ellipsoidal corrections.

The Stokes function, $S(\psi)$, can then be used to solve the geoid heights above the geodetic ellipsoid [Ewing, 1976].

$$N_g = \frac{1}{4\pi\, \gamma_m} \int_0^{2\pi} \int_0^{\pi} \Delta g(\psi,\alpha) S(\psi) \sin\psi \, d\psi \, d\alpha$$

where

γ_m : the mean value of normal gravity

ψ : the angular distance between the point where N is being determined and the area where the effect of Δg is being considered

α : the azimuth from the affected point to that causing the effect

Δg : the gravity anomaly

$S(\psi)$: the Stokes function

The Stokes function is given by [Ewing, 1976]

$$S(\psi) = \csc\frac{\psi}{2} + 1 - 6 \sin\frac{\psi}{2} - 5 \cos\psi - 3 \cos\psi \ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right)$$

The gravitational potential using degree 360 and order 360 has been tested through comparison of Doppler station geoid heights with geoid heights from Rapp's versions of geopotential models. The agreement between the two geoid height measurements is approximately ± 1.6 m [Rapp, 1986].

III. GEOID HEIGHT FROM GPS

A. THE GLOBAL POSITIONING SYSTEM

The Global Positioning System (GPS) calls for a precise navigation system divided into three segments: space segment, control segment and user equipment. The space segment will consist of three orbital planes of satellites at inclinations of 120° in circular orbits at altitudes of 20,000 km. Each plane will eventually contain six to eight satellites to give the three dimensions of position, velocity and precise time 24 hours a day anywhere in the world. The control segment consists of the ground stations necessary to track the satellites, monitor the system operation, and periodically provide corrections to the navigation and time signals. Each satellite broadcasts signals containing information on its position. The GPS satellite transmits signals at two L-band frequencies (1227 and 1575 MHZ) to permit corrections for ionospheric corrections. The signals are modulated with two codes: P, which provides for precise measurement, and C/A, which permits easy lock-on to the desired signal. The user segment consists of the equipment necessary to convert the satellite navigation message into useful navigation information.

The navigation message contains the data that the user's receiver requires to perform the operations and computations for successful navigation with GPS. The data includes: (1) information on the status of the Space Vehicle (SV); (2) time synchronization information for the conversion of the C/A to P code; (3) parameters for computing the clock correction and the ephemeris of the SV; and (4) corrections for delays in the propagation of the signal through the atmosphere. In addition the data contain almanac information to define the approximate ephemeris and to give the status of all the other SV information which is required for use in signal acquisition [Milliken, 1980]. Ranges to the satellites are determined by signal transit times multiplied by the speed of light (299,792,458 m/sec). The transmitted message contains ephemeris parameters that enable the user to calculate the position of each satellite at the time of the transmission of the signal.

B. CARRIER PHASE MEASUREMENTS

GPS measurements can be made using the pseudo-range and the carrier phase. The pseudo-range is essentially a measurement of distance contaminated by clock error. When four satellites are observed simultaneously, the three dimensional position of the ground receiver can be determined along with the receiver clock offset at a single epoch. The accuracy of pseudo-ranges is affected by multipath effects, which depend on the antenna design, and its height above the ground.

Carrier phase measurements are more precise than pseudo-range and are not as vulnerable to multipath effects. They can be used to compute the precise base line components ΔX, ΔY, ΔZ between two receivers. Phase measurements are made by beating the received carrier with the signal from a local oscillator internal to the GPS receiver. The slant range from a GPS receiver to a satellite can be modelled in terms of time. It takes the signal time to travel between the satellite and the receiver or the equivalent number of cycles. The range of the cycles will consist of an integer and fractional number of cycles. When a receiver locks onto the carrier signal, it can immediately measure the fractional part and begin counting subsequent integer cycles, but it can not measure or account for the initial integer number of cycles that preceded the initial fractional part. The initial integer ambiguity which biases the subsequent measurements is called the initial integer ambiguity bias.

GPS uses a one-way carrier beat phase. The GPS satellite and receiver are controlled by separate clocks. The satellite clock generates the signal, and the receiver clock detects when the signal arrives. An error in the synchronization of the clocks of 1 microsecond creates an error in range of 300 m.

C. ONE-WAY CARRIER PHASE MEASUREMENT DIFFERENCING

A single difference, SD(j,i), is formed by differencing carrier beat phase observables from two receivers 1 and 2 at the same observation epochs i of same satellite j. The equation is given by [Remondi, 1984]:

$$SD(j,i) = S(2,j,i) - S(1,j,i)$$

where S(k,j,i) is the raw, unpreprocessed, fractional phase plus the count made at epoch i by receiver k for satellite j. The main advantage of the single difference is that it reduces or eliminates satellite orbital and clock errors, because they are common to both receivers. Its disadvantage is that one can not exploit the integer nature of integer ambiguities. Thus, for short base lines, the ultimate in accuracy may not be achievable [Remondi,1985].

A double difference, DD(j,k,i), is formed by differencing single differences between a reference satellite j and another satellite k at the same epoch i. The equation is given by [Remondi, 1984]:

$$DD(j,k,i) = SD(k,i) - SD(j,i)$$

The advantage of the double differences is that the receiver clock dependent terms are eliminated because the differences for each epoch are correlated. The significance of the removal of those terms is to reduce from nanoseconds to microseconds the timing accuracy required to achieve one cycle accuracy. For short base lines, the integer ambiguities can be isolated, since the contribution made by the clock drift is reduced [Remondi, 1985]. The Trimble 4000SX receiver achieves sub-microsecond accuracy by using the C/A code timing information [Ashjaee, 1985].

A triple difference, TD(j,k,i), is formed by differencing the double differences for the same satellite pair at some integer of succeeding epochs i+1 [Remondi, 1984].

$$TD(j,k,i) = DD(j,k,i+1) - DD(j,k,i)$$

The advantage of the triple difference is that it eliminates all the time independent terms, namely the initial integer ambiguities, and becomes insensitive to the initial ambiguities and any cycle slips when the receiver loses lock. The disadvantage of the triple difference is, another level of correlation, loss of resolution and a greater number of observations. Triple differences are already correlated with respect to satellite because of the underlying double differences and are further correlated with respect to time because consecutive triple observations will have common DD(j,k,i+1) terms.

For short base lines local area integer ambiguities can easily be resolved because unmodelled errors are highly correlated between the two antenna sites and are mostly eliminated by differencing. Algorithms can take advantage of the integer nature of the initial ambiguities and solve for them [Remondi, 1984].

D. GEOID HEIGHT FROM GPS

Surface fitting techniques can be used with GPS-derived geoid heights. GPS stations are likely to be close together, of the order of a few tens of km, and the local geoid can be estimated directly if levelling data is available in the area. This together with it's relative simplicity makes the method practical for correcting GPS heights.

It is possible to use the two sets of elevations (that is, levelled and GPS-derived) to define two distinct planes. The published levelled elevations are referred to the geoid and the GPS elevations are referred to an ellipsoidal surface. If both elevations are made equal at one bench mark (control station), then in general, the other bench marks will have two elevation values. After several different models were studied two mathematical surface models were chosen in this study. The five-parameter model found suitable for use on the NPS campus is

$$N_0(X,Y,Z) = h_0 + \Delta h - H + a\Delta Y + b\Delta X^2 + c\Delta Y^2 + d\Delta X\Delta Y$$

and the six-parameter model selected for a larger area near Monterey, California, is given by

$$N_0(X,Y,Z) = h_0 + \Delta h - H + a'\Delta X + b'\Delta Z + c'\Delta X^2 + d'\Delta Y^2 + e'\Delta X\Delta Y$$

where

$N_0(X,Y,Z)$: the global geoid height, obtained by gravity measurements

h_0 : the ellipsoid height of the reference point, including a constant correction to $N_0(X,Y,Z)$ at the reference station

Δh : the ellipsoid height difference with respect to the reference station

H : the published or levelled elevations at each control station

a, b, c, d, a', b', c', d', e' : the coefficients to be determined

ΔX, ΔY, ΔZ : the coordinate differences in WGS 84

These models can be solved to determine the east-west and north-south tilts which are absorbed by two rotations, one around the north axis (ΔY) and the other around the east axis (ΔX) in the horizon system. Their separation is absorbed by the scale correction [Zilkoski, 1988]. The local geoid heights, N, can be found from the equation

$$N = N_0(X,Y,Z) + \Delta N$$

where ΔN is the variation of geoid height in local area. The equation is given by

$$\Delta N = H + N_0(X,Y,Z) - \Delta h$$

$$\Delta N = h_0 + a\Delta Y + b\Delta X^2 + c\Delta Y^2 + d\Delta X\Delta Y$$

or

$$\Delta N = h_0 + a'\Delta X + b'\Delta Z + c'\Delta X^2 + d'\Delta Y^2 + e'\Delta X\Delta Y$$

To solve for h_0, a, b, c, d, a', b', c', d' and e', the global geoid heights $N_0(X,Y,Z)$ can be obtained from the Geoid.exe program described in Chapter IV. The geoid model can be rearranged to give

$$H = h_0 + \Delta h - N_0(X,Y,Z) + a\Delta Y + b\Delta X^2 + c\Delta Y^2 + d\Delta X\Delta Y$$

or

$$H = h_0 + \Delta h - N_0(X,Y,Z) + a'\Delta X + b'\Delta Z + c'\Delta X^2 + d'\Delta Y^2 + e'\Delta X\Delta Y$$

Then h_0, a, b, c, d, a', b', c', d' and e' can be solved by the least squares method. The geoid height, N, found by this method appears to be adequate for areas up to 50 km x 50 km where the geoid is smooth [King, 1985].

E. METHOD OF COLLOCATION

The method of collocation was derived from least squares interpolation. This method was used to predict the geoid height, N, for a local area. The geoid model is given by

$$N = N_0(X,Y,Z) + \Delta N + S + n$$

where

S : the signal

n : the noise

$N_0(X,Y,Z) + \Delta N$: the system function from the surface models at control points where both h and H are known

Then, for a given $N_0(X,Y,Z)$ the method of collocation can be used to determine S_q at these control points and to predict S_p at other points in the local area. At any point in the local area the value of h can be determined by using differential GPS measurements between a known point and any other point by the equation

$$h = h_0 + \Delta h$$

The value of H at any point is given by

$$H = h - N_0(X,Y,Z) - \Delta N - S - n$$

The value of H at any point in the local area, which depends on the value of h at the control points and the differential GPS measurements, can be predicted with an accuracy of ± n. The general form of the observation equation in the method of collocation is [Jeyapalan, 1977]:

$$x = A \bullet X + S_q + n_q + \mathbf{0} \bullet S_p$$

where

x : the vector of the observation ($x = \Delta h - N_0(X,Y,Z) - H$)

A : a given rectangular coefficient

X : the vector of the systematic parameters (h_0, a, b, c, d, or h_0, a', b', c', d', e')

S_q : a signal vector at q observation points

n_q : a vector of measuring errors, noise at q points

S_p : a signal vector at p unknown stations

$\bullet$: indicates matrix multiplication

$\mathbf{0}$: the null matrix

If

$$Z_q = S_q + n_q$$

Then

$$x_q = A \bullet X + Z_q + 0 \bullet S_p$$

then

$$X = (A^T C_q^{-1} A)^{-1} A^T C_q^{-1} x$$

and

$$S_p = C_{pq} C_q^{-1} (x - A X)$$

where the variance-covariance matrix C of the Z_q and S_p vectors is

$$C = \begin{pmatrix} C_p & C_{pq} \\ C_{pq} & C_q \end{pmatrix}$$

The essence of this method is that by some means a covariance matrix can be assigned to the signal. For noise it will be possible to assign a diagonal weight matrix.

S_p are the values of the signal at the interpolated stations. Suppose there are q observations (and values of S_q), p interpolated values of S_p and m model parameters. The covariance matrices are the following [Bomford, 1980]:

(i) C_q, the expected covariance between the observed x's for all pairs of the q observations. It is a q x q matrix.

(ii) C_{sq}, the expected covariance between the signals for all pairs of the q observations. It is a q x q matrix.

(iii) C_{nq}, the expected value of n^2 at each station. It is a q x q diagonal matrix.

(iv) C_{pq}, the expected covariance between all pairs of mixed observed and interpolated signals. It is a p x q matrix.

(v) C_p, the expected covariance between the signals at pairs of interpolated stations. It is a p x p matrix.

The variance of the noise, C_{nq} can be estimated in the usual way according to the circumstances at each station, different types of instrument, etc. The noise at different points is independent; hence

$$C_{nq} = \begin{pmatrix} \sigma_n & 0 \ldots\ldots 0 \\ 0 & \sigma_n \ldots\ldots \\ \ldots\ldots & \ldots\ldots \\ 0 & 0 \ldots\ldots \sigma_n \end{pmatrix}$$

where σ_n is the standard error of the noise.

The expected covariance between the observation x's, C_q can be obtained by

$$C_q = C_{nq} + C_{sq}$$

because the signal is small.

The C_{sq} can be be computed from a simple function whose parameters can be determined by using empirical data. The covariance function is positive and definite. These two characteristics are found in many functions. Functions commonly used for covariance may be constant, sinusoidal, Gaussian, exponential, exponential cosine, and exponential sine and cosine. In this thesis the sinusoidal function was used. The function is given by

$$C_{sq} = B \sin(kr)$$

$$C(r) = C_{nq} + B \sin(kr)$$

where C_{nq} is the standard deviation of the control stations and B, k are coefficients to be determined.

The parameters can be determined by least squares, and residuals at each point can be computed. From the residual, the covariance between points at a distance (r) can be computed by

$$C(r) = \frac{\sum_{i=1}^{n} V_0 V_r}{n - 1}$$

where V_0 is the residual at the center, V_r is the residual at a point which is at a distance r from the center and n is the number of points. There are several methods of determining C(r) of which the concentric circle approach is the

simplest. The coefficients B and k can then be computed from the computed covariance.

The expected covariance between the signals at pairs of interpolated stations C_p can be obtained by substituting the distance between the interpolated stations and control stations into the covariance function.

The expected covariance between all points of mixed observed and interpolated signals C_{pq} can be obtained from the covariance function for each distance from the interpolated stations to the control stations.

In this thesis two cases were studied in solving for the geoid height model.

Case 1. $C_q = \mathbf{I}$ and $C_{pq} = 0$

Case 2. $C_q \neq \mathbf{I}$ and $C_{pq} \neq 0$

I start with assuming

$$C_q = P^{-1} = \mathbf{I}$$

$$C_{pq} = 0$$

where P is the weight matrix and **I** is the unit matrix.

Then

$$X_0 = (A^T P A)^{-1} A^T P x$$

and

$$S_p = 0 \bullet P (x - A X_0) = 0$$

Using X to compute residuals, v, and then estimating C_q, C_{pq} and P, it is then found that

$$X_1 = (A^T C_q^{-1} A)^{-1} A^T C_q^{-1} x$$

and

$$S_p = C_{pq} C_q^{-1} (x - A X_1)$$

IV. DATA COLLECTION AND INSTRUMENTATION

A. PLANNING

Nine temporary bench marks were established on the NPS campus (Figure 4). Bench marks GH7 and GH8, designated as check marks, were established in the center of the levelling loop. H of bench marks was obtained by differential levelling. Δh was measured with a pair of GPS receivers.

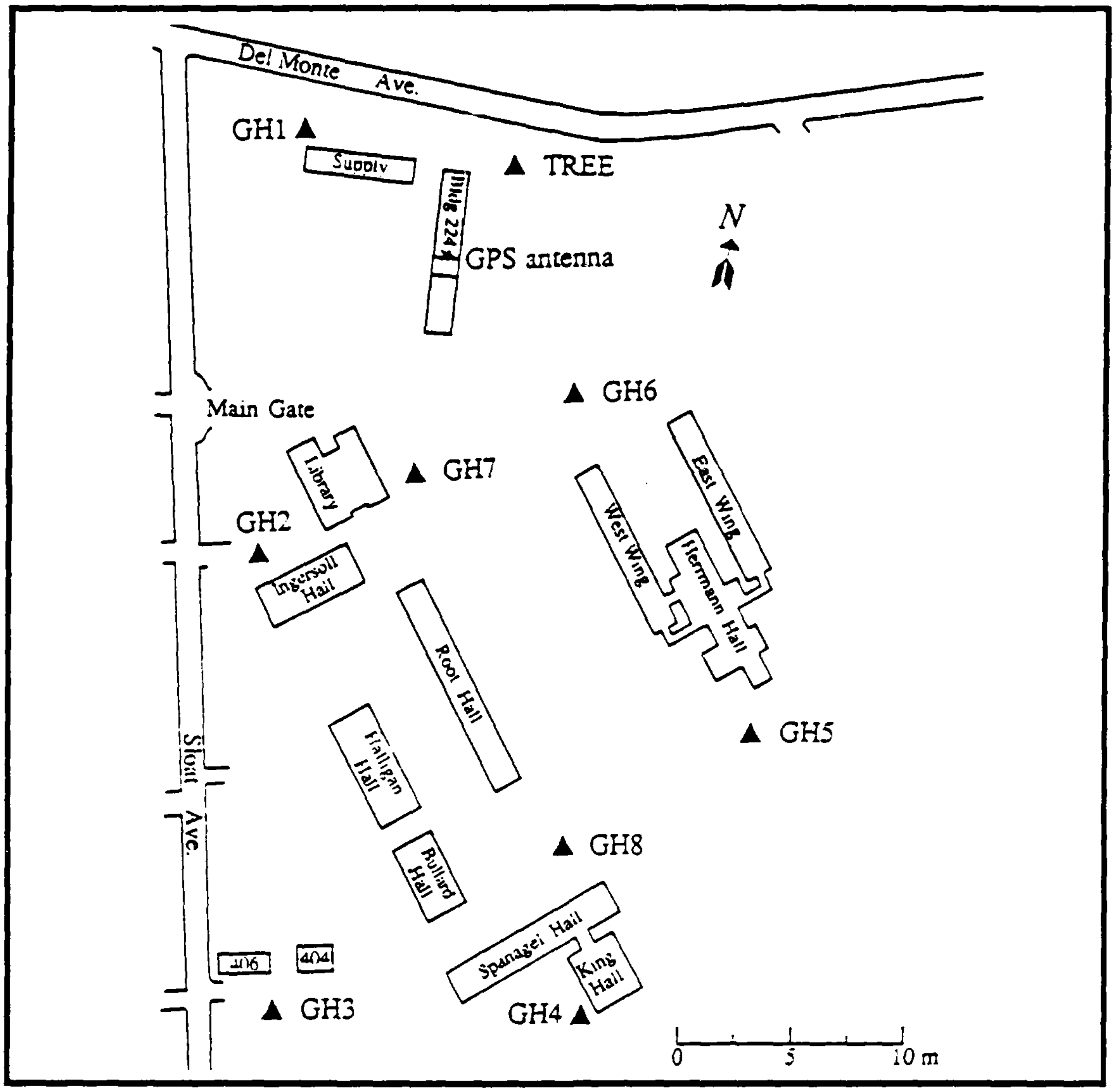

Figure 4. The temporary bench marks on the NPS campus.

Eight permanent bench marks were recovered in the study area (Figure 5) [Vertical Control Data, 1961]. Bench mark GWM 27, designated as a check mark, was roughly in the center of the survey area for the permanent bench marks. H was obtained from the published elevations. Δh was also measured with a pair of GPS receivers.

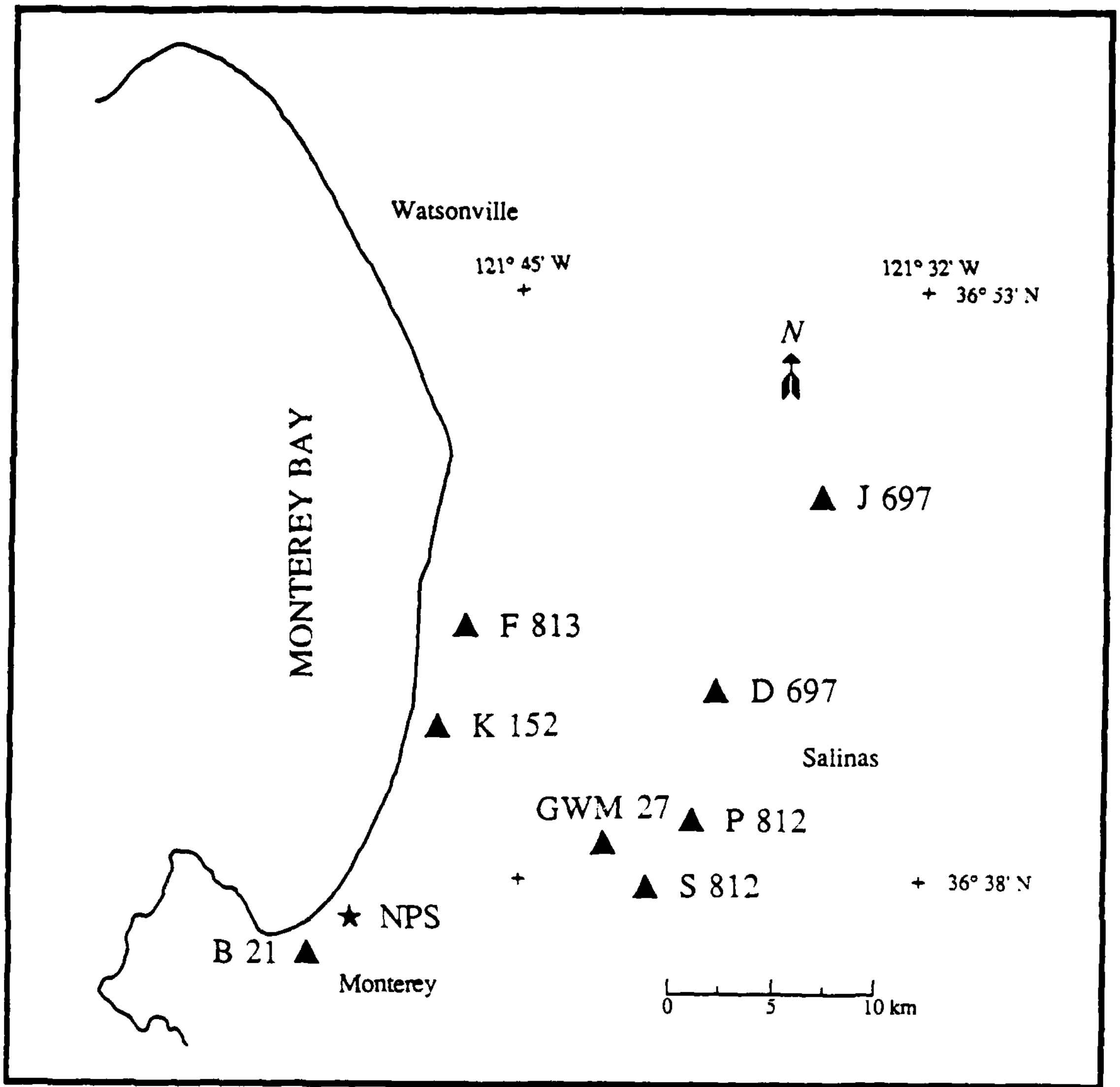

Figure 5. The permanent bench marks in the Monterey Bay area.

B. DATA COLLECTION

1. Obtaining H

a. Precise Levelling

Precise levelling was run on the NPS campus on February 19, 1988, and March 24, 1988. The elevation of station TREE was assumed to be zero above the MSL, so the orthometric height differences, ΔH, for each temporary bench mark could be observed. The differences of the relative elevations, ΔH, for the forward and the backward sights on the NPS campus are listed in Table 1.

Table 1. ΔH FOR THE NPS CAMPUS

Bench mark		ΔH (m)		
From	To	Forward	Backward	Mean
TREE	GH1	-0.808	0.808	-0.808
GH1	GH2	5.388	-5.384	5.386
GH2	GH3	3.150	-3.149	3.149
GH3	GH4	1.535	-1.536	1.535
GH4	GH5	-1.016	1.016	-1.016
GH5	GH6	-4.214	4.215	-4.214
GH6	GH7	0.489	-0.489	0.489
GH7	TREE	-4.031	4.033	-4.032
GH6	GH2	0.547	-0.546	0.547
GH7	GH2	0.057	-0.057	0.057
GH3	GH8	0.716	-0.718	0.717
GH8	GH4	0.817	0.817	0.817

To avoid an obstruction near bench mark D 697, which is 10 ft north of a 12-inch cedar tree, the temporary bench mark TEMP 1 was established nearby. Offset levelling was run on April 8, 1988. The elevation of TEMP 1 was offset by -0.401 m from the elevation of D 697.

F 813 which was set in the west end of the south abutment of a steel bridge over the Salinas River was offset to temporary bench mark TEMP 2. The offset levelling was run on April 9, 1988. The elevation of TEMP 2 was offset by -2.218 m from the elevation of F 813.

b. Published Elevation

H values for the larger of our two study areas in and near Monterey, California, areas were obtained from the published elevations printed by U.S. Department of Commerce Coast and Geodetic Survey Washington D.C. [1961]. The geodetic datum used in this publication was the NGVD 29. First-order spirit levelling has extended this datum over most of the continent. Although first-order lines may be 300 km apart in some western areas, most points in the country are no more than 50 km from an estimated first-order bench mark. A readjustment of the this network is the NAVD 88. This new adjustment will be made to the geopotential surface rather than to sea level, and it will place the existing vertical data in a form that makes it most consistent and accessible to the user. Changes to older published elevations are not expected to exceed 15 decimeters [NASA, 1978]. Table 2 lists the orthometric heights of the permanent bench marks we used.

Table 2. H FOR PERMANENT BENCH MARKS UESD IN THIS STUDY

Bench mark	H (m)
K 152	16.965
B 21	5.787
S 812	15.978
P 812	13.134
D 697	30.057
J 697	85.061
F 813	7.983
GWM 27	63.487

2. Obtaining Δh

a. Satellite Observation Plan

Due to the positions of the satellite orbits during this study, the observing window was between 2100 and 0200 hours Pacific Standard Time (PST=UTC+8 hours.). The time period was between February 2, 1988, and May 12, 1988.

b. Satellite selection

The same five satellites (SV) (6, 9, 11, 12 and 13) were used for all observations.

c. Position Dilution of Precision (PDOP)

The accuracy to which positions are determined using GPS depends on two factors: (i) satellite configuration geometry, and (ii) measurement accuracy. GPS measurement accuracy represents the combined effect of ephemeris uncertainties, propagation errors, clock and timing errors, and receiver noise.

The effect of satellite configuration geometry is expressed by the dilution of precision (DOP) factor, which is the ratio of the positioning accuracy to the measurement accuracy [Wells, 1987].

$$\sigma = DOP \bullet \sigma_0$$

where

σ_0 : is the measurement accuracy (standard deviation)

σ : is the positioning accuracy (standard deviation in one coordinate)

The value of GDOP itself is a composite measure that reflects the influence of satellite geometry on the combined accuracy of the estimation of observation time (user clock offset) and receiver position [Milliken, 1986].

$$GDOP^2 = PDOP^2 + TDOP^2$$

TDOP is the Time Dilution Of Precision, the error in the clock of the receiver bias multiplied by the velocity of light. The four best satellites selected by the receiver are those with the lowest GDOP. Trimble recommends that the rapidly changing PDOP provides better geometry for phase differencing techniques. Low or constant PDOP provides weaker solutions. The 4000SX receiver does not record GDOP, but it does record PDOP every five minutes. PDOP was about 4.5 for all GPS observations.

C. INSTRUMENTATION

1. Levelling

A Zeiss model Ni-2 level (Ser. # 82377) was used at the temporary bench marks for the third-order, class I levelling. It has a 32-diameter magnification, produces an erect image, and has stadia constants of 333 or 100. A Peg test was performed before levelling. The level error, or c-value was also checked before the beginning the levelling. The c-value was -0.006 mm/m which was less than +0.05 mm/m, so it was not necessary to adjust the level [Bodnar, 1975]. The level contains a bubble tube to permit positioning parallel to the geoid. When properly set up at a point, the telescope is locked so that it will rotate through a 360° arc in a horizontal plane. With the level locked in position readings are made on two calibrated staffs held in upright positions ahead of and behind the instrument. The difference between readings is the difference in elevation between points. Dietzgen Model # 6450 metric rods were used in the levelling. These rods are graduated in centimeters. The actual reading is estimated to the nearest 1 mm. Rod levels were used to indicate when were vertical.

2. GPS

a. Trimble 4000SX Receiver

A complete description of the Trimble 4000SX receiver is given by Trimble Navigation [Trimble, 1987a]. NPS operates three Trimble 4000SX GPS receivers. For this study I fixed one antenna to the roof of Building 224 on the NPS campus, and one was carried to the field. The 4000SX is capable of observing the C/A code, integrated Doppler, and carrier beat phases of up to five satellites simultaneously. Its ability to use the C/A code allows the receiver to be used as a stand alone navigation system to determine positions using Doppler-smoothed pseudoranges and velocities [Ashjaee, 1985]. The receiver uses the C/A code in a time transfer mode to determine the offset and drift of its own clock and thus provide accurate time tags for the observations without the requirement of an external atomic clock or synchronization with the receiver at the other end of the baseline. The reference position (the geodetic coordinates of the antenna) and the practical options chosen must be entered into the receiver via the receiver key pad. The 4000SX requires 115 V AC power or 12 V DC

power supply. 115 V AC power was used for the NPS campus measurements. 12 V DC, supplied by a car battery, was used in the field.

b. Grid Personal Computer (PC)

For precise relative positioning the 4000SX receiver transmits data through an RS-232 port to a microcomputer (Grid PC) for storage on floppy disks for post-processing. The Grid PC uses either 115 V AC or 12 V DC. On the NPS campus a 115 V AC power supply was used, so measurements could be made for four hours. The 12-volt battery in the Grid PC lasts about 120 minutes, so measurements were taken for only 100 minutes in the field.

c. Antennas

Multipath-resistant Trimble microstrip antennas were installed on Building 224 on the NPS campus and over the various bench marks. The antenna heights from the center of bench marks to the edge of the antenna's ground plane were measured before and after GPS observations. The field antenna was mounted on a tripod with a tribach and optical plummet for centering and levelling the antenna. Arrows on the antennas were oriented to the north at both stations using a magnetic compass.

d. Meteorological Instruments

A barometer and a sling psychrometer were used to measure atmospheric pressure, relative humidity and air temperature at each field site.

e. The steel tape

A three-meter steel tape was used to measure the antenna height. This tape can be read to 1 cm. Readings are estimated to 1 mm.

D. SOFTWARE

A complete description of the Trimble-supplied Trimvec software is to be found in Trimble Navigation [Trimble, 1987b]. The software provides data logging, baseline computation and datum transformation programs. These operate with an IBM compatible personal computers. A printer and a hard disk are used with the planning and processing programs. The following programs are used on 3-1/2 inch micro-floppies:

1. Data Logger on Disk 1

In relative positioning, the receiver is controlled from the Grid PC by version D of Trimble's Gridlog5.bat program. Each observation session was initialized to log data when a minimum of four satellites were 15° above the

antenna's horizon. Five satellites were designated for each observing session. The observables and receiver clock parameters were logged to a floppy disk every 15 seconds and the C/A code-determined antenna position and PDOP every five minutes. The GPS navigation message was logged to a separate file at the beginning of the session.

2. Post-processor on Disk 2

The data was processed using the Trimvec Trim640 program, Revision AB. Trim640 is a relative positioning, post-processing program that provides triple and double difference solutions for two sets of the 4000SX carrier phase data logged simultaneously at two stations. Trim640 adopts the best C/A code position during the data loading. Only the broadcast ephemeris can be used to compute fixed orbit satellite positions. Trim640 limits processing to 700 epoches, so the first 700 epoches for each observing session are used. The antenna on the roof of Building 224 on the NPS campus was used as reference station and its coordinates were kept fixed. Permanent bench marks were chosen as Trim640 reference stations for the off campus area because the observing sessions at them were shorter than the observing session at Building 224 on the NPS campus.

3. Geoid.exe on Disk 3

This program computes global geoid height, $N_0(X,Y,Z)$, at any point with an accuracy of a few meters [Trimble, 1987b]. The global geoid heights, $N_0(X,Y,Z)$, are based on Rapp's 1978 360 x 360 model, an harmonic expansion of the geopotential referred to GRS 80. Heights are suitable for showing relative shape and trends in geoid. Global geoid height for the Monterey Bay area from 36° 34' N to 36° 49' N latitude, and from 121° 34' W to 121° 55' W longitude is given in Figure 6. A Fortran program GPSCON (Appendix A) was used for the contoured plot.

A second program also named Geoid.exe was obtained from the NGS of NOAA's National Ocean Service Charting and Geodetic Services uses the same Rapp's 1986 360 x 360 model. The global geoid height in the Monterey Bay area from the NGS program is shown in Figure 7. Because the NGS program rounds off to the nearest decimeters, there are some steps on the curves. The differences between Figure 6 and 7 may be attributed to differences between Rapp's 1978 and 1986 models.

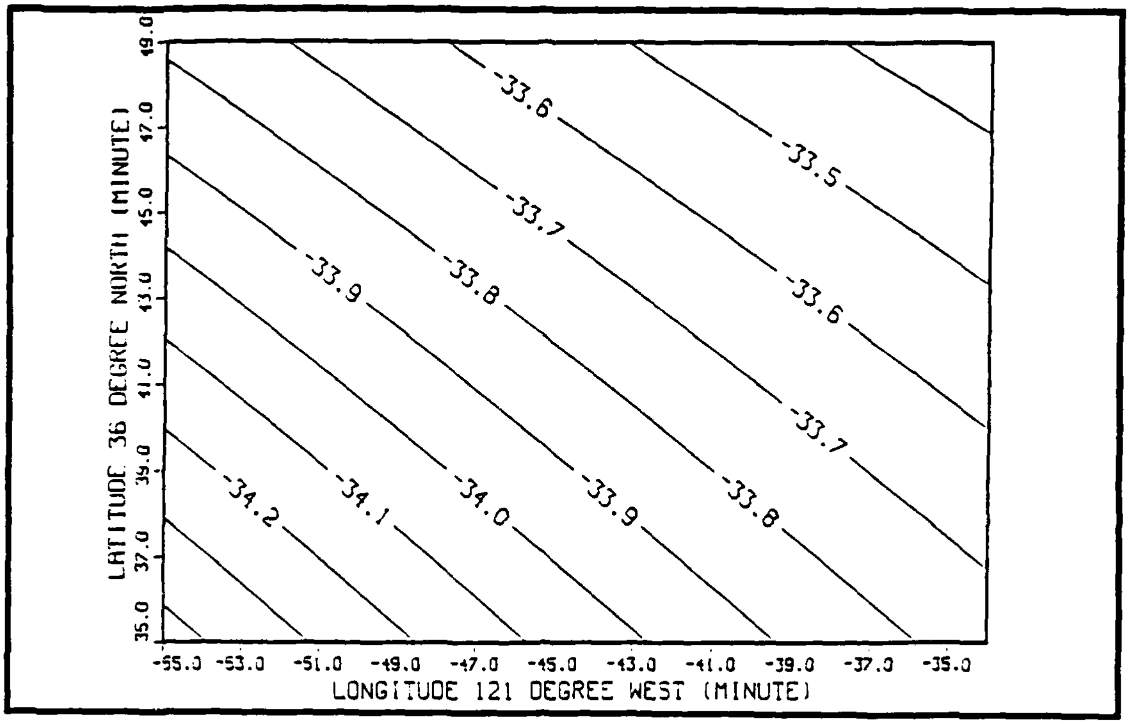

Figure 6. The global geoid height in the Monterey Bay area calculated using the Trimvec Geoid.exe program.

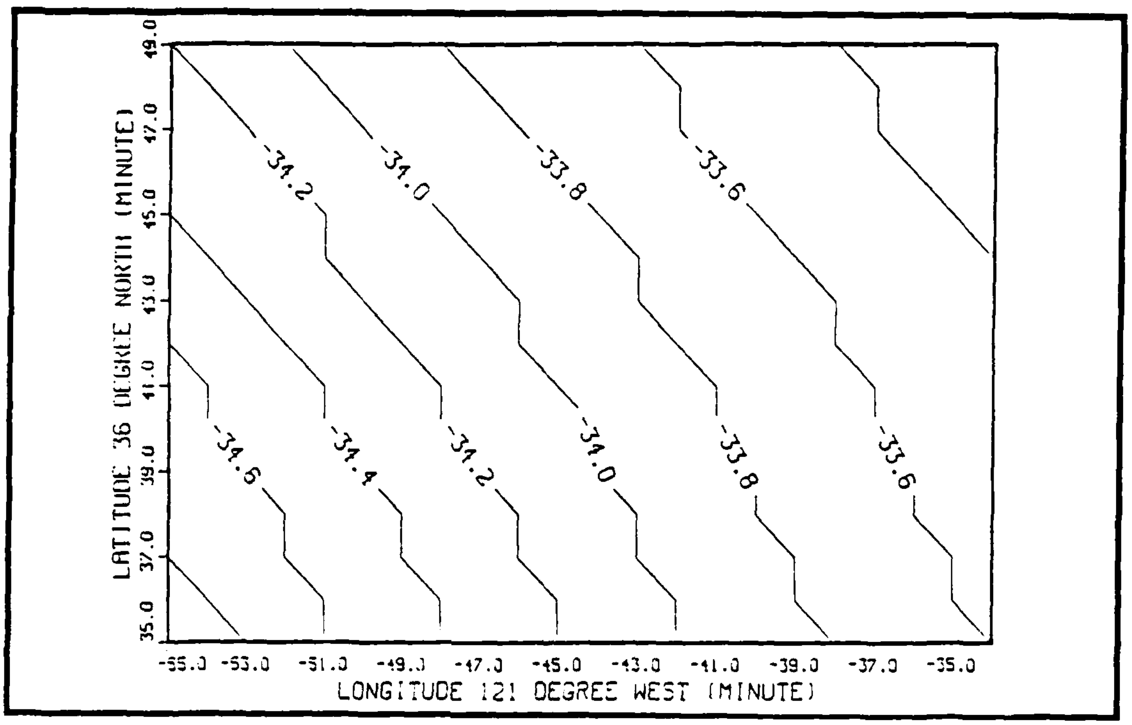

Figure 7. The global geoid height in the Monterey Bay area calculated using the NGS Geoid.exe program.

4. <u>Satellite Visibility Program on Disk 3</u>

The program Stvis.exe is a system for generating visibility data for the GPS satellites. The program takes as input the observer's latitude, longitude, a mnemonic name for the location, data for calculations and optionally the time zone offset from Universal Time Coordinates (UTC). NPS in Monterey, California, was used as the reference station (36° 35' 56.1" N, 121° 52' 36.0" W, and altitude -19 m). Satellite orbit data is contained in the file, Almanac.dat, which is produced by processing data collected from the actual satellites. The satellite visibility chart shows the tracks of the GPS satellites during the planned observation session (Figure 8).

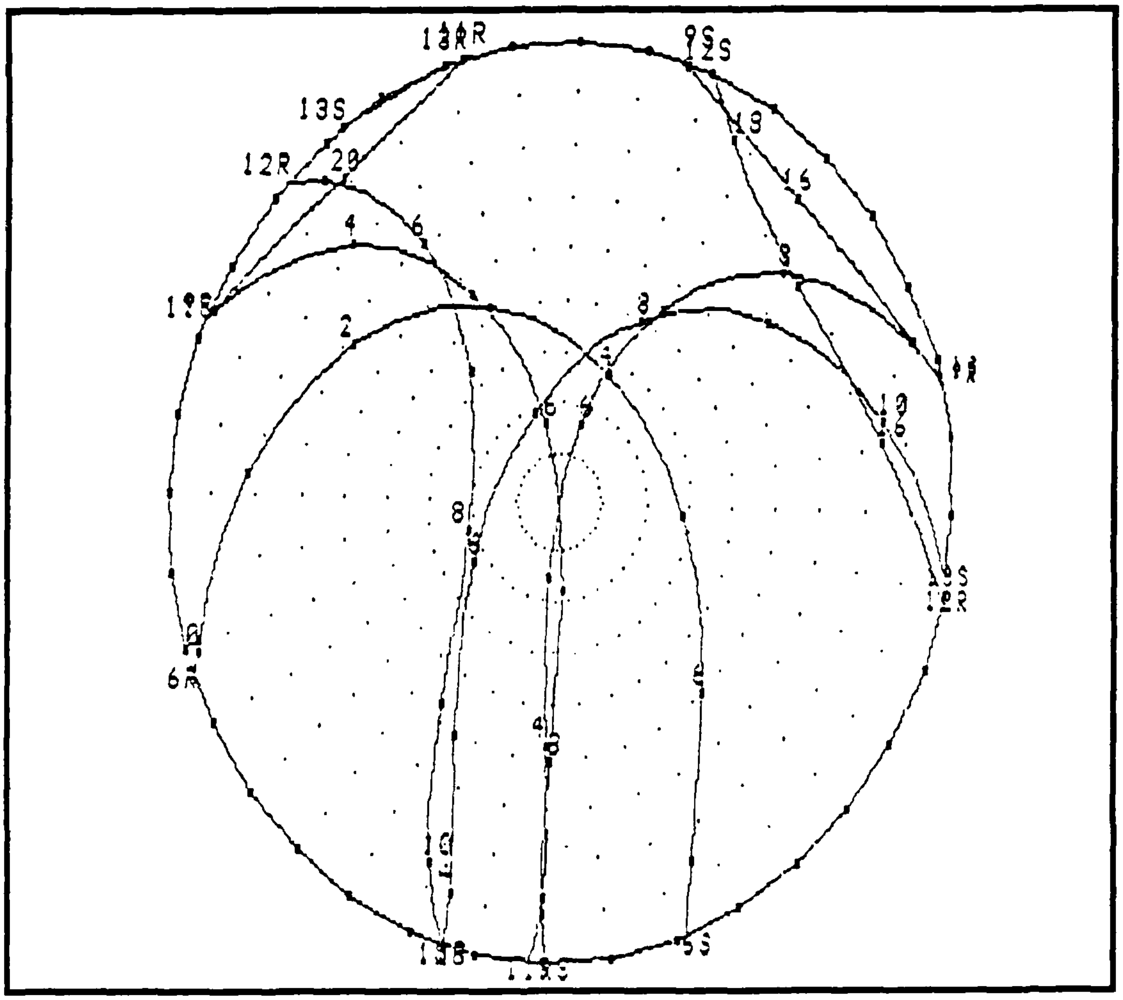

Figure 8. Sky plots of satellite tracks for the Monterey Bay area. Elevation angles are dotted concentric circles. Zenith is at the center.

V. DATA PROCESSING

A. ORTHOMETRIC HEIGHT ON NPS CAMPUS

In calculating the most probable elevations for the temporary bench marks, station TREE was used as a reference, where the elevation was assumed to be 0 meters above MSL. The interlocking levelling circuit on the NPS campus is shown in Figure 9.

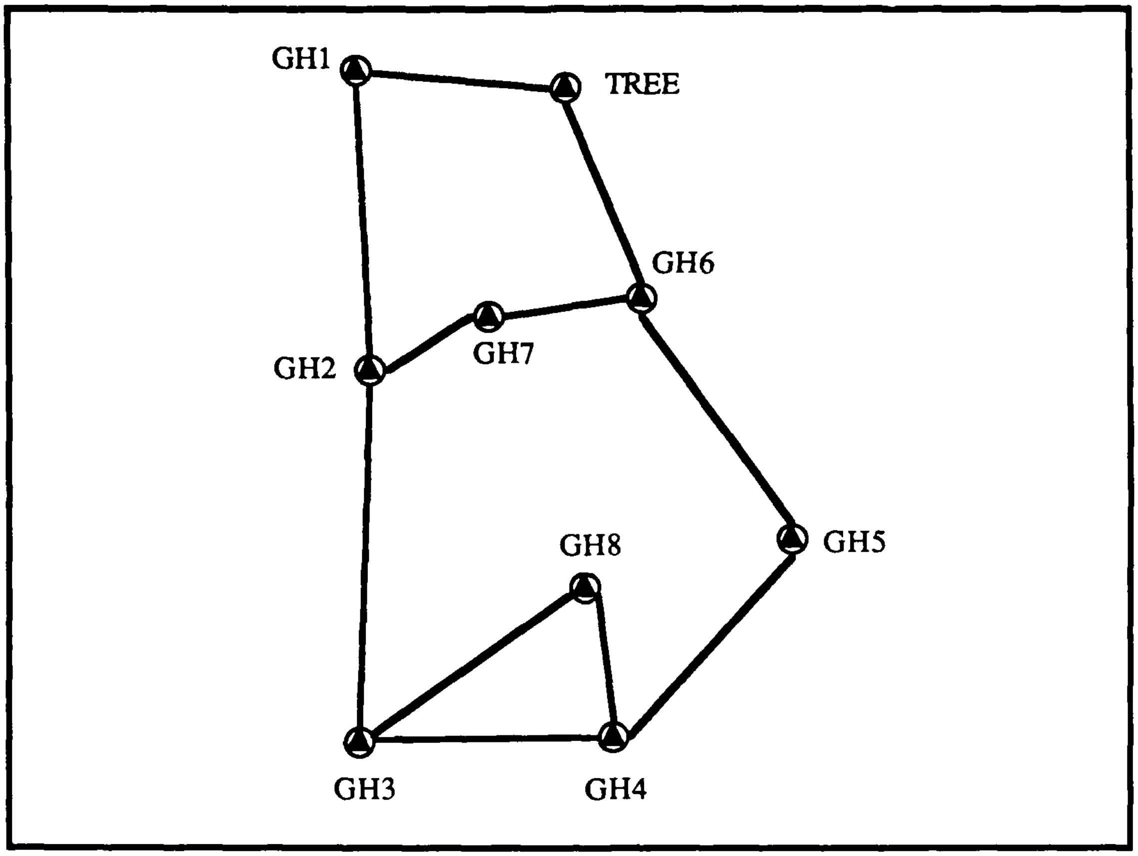

Figure 9. The interlocking levelling circuit on the NPS campus.

A constrained least squares adjustment was used to do the computation. A weight of 100 was assumed at the TREE and weights of 1 were used for all observations. The 13 observation equations are:

$$
\begin{aligned}
P_1\, H_{TREE} &= P_1 (\ 0.000 + V_1\ ) \\
P_2 (H_1 - H_{TREE}) &= P_2 (\ -0.808 + V_2\ ) \\
P_3 (H_2 - H_1) &= P_3 (\ 5.386 + V_3\ ) \\
P_4 (H_3 - H_2) &= P_4 (\ 3.149 + V_4\ ) \\
P_5 (H_4 - H_3) &= P_5 (\ 1.535 + V_5\ ) \\
P_6 (H_5 - H_4) &= P_6 (\ -1.016 + V_6\ ) \\
P_7 (H_6 - H_5) &= P_7 (\ -4.214 + V_7\ ) \\
P_8 (H_7 - H_6) &= P_8 (\ 0.489 + V_8\ ) \\
P_9 (H_{TREE} - H_7) &= P_9 (\ -4.032 + V_9\ ) \\
P_{10} (H_2 - H_6) &= P_{10} (\ 0.547 + V_{10}) \\
P_{11} (H_2 - H_7) &= P_{11} (\ 0.057 + V_{11}) \\
P_{12} (H_8 - H_3) &= P_{12} (\ 0.717 + V_{12}) \\
P_{13} (H_4 - H_8) &= P_{13} (\ 0.817 + V_{13})
\end{aligned}
$$

The observation equations in matrix form are [Wolf, 1980]:

$$\mathbf{P\,A\,H} = \mathbf{P\,L} + \mathbf{PV}$$

The weight matrix $\mathbf{P}_{13x13}$, coefficient matrix $\mathbf{A}_{13x9}$, observation matrix $\mathbf{L}_{13x1}$, and residual matrix $\mathbf{V}_{13x1}$ are:

$$
\mathbf{P} = \begin{pmatrix}
100 & & & & & & & & & & & & \\
& 1 & & & & & & & & & & & \\
& & 1 & & & & & & & & & & \\
& & & 1 & & & \text{ZEROS} & & & & & & \\
& & & & 1 & & & & & & & & \\
& & & & & 1 & & & & & & & \\
& & & & & & 1 & & & & & & \\
& & & & & & & 1 & & & & & \\
& & & & & & & & 1 & & & & \\
& & & & & & & & & 1 & & & \\
& & \text{ZEROS} & & & & & & & & 1 & & \\
& & & & & & & & & & & 1 & \\
& & & & & & & & & & & & 1
\end{pmatrix}
\qquad
\mathbf{A} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
\mathbf{L} = \begin{pmatrix} 0.000 \\ -0.808 \\ 5.386 \\ 3.149 \\ 1.535 \\ -1.016 \\ -4.214 \\ 0.489 \\ -4.032 \\ 0.547 \\ 0.057 \\ 0.717 \\ 0.817 \end{pmatrix}
\qquad
\mathbf{V} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \\ V_{10} \\ V_{11} \\ V_{12} \\ V_{13} \end{pmatrix}
$$

The program Lobs.basic [Jeyapalan, 1988] was used to find H for the temporary bench marks. The program (Appendix B) gives the **H** and **V** matrices:

$$H = \begin{pmatrix} 0.000 \\ -0.808 \\ 4.579 \\ 7.728 \\ 9.262 \\ 8.246 \\ 4.032 \\ 4.521 \\ 8.445 \end{pmatrix} \qquad V = \begin{pmatrix} 0.00000 \\ 0.00007 \\ 0.00007 \\ -0.00022 \\ -0.00044 \\ -0.00022 \\ -0.00022 \\ 0.00007 \\ -0.00019 \\ -0.00010 \\ -0.00010 \\ 0.00022 \\ 0.00023 \end{pmatrix}$$

The standard deviation of the observation equations is 0.36 mm.

B. GPS DATA PROCESSING

1. Automatic Processing Mode

Trimble recommends that an automatic processing mode be used if more than one hour of data from four or five GPS satellites is taken at each site for baselines lengths up to 30 km.

The output files contain triple difference, double difference float and double difference fixed solutions. The double difference fixed solution provides the most accurate solution if the following conditions are met:

a. Integer bias search indicates the ratio sum-of-squares must be greater than 3.0;
b. RMS (cycles) less than 0.05; and
c. Difference between the float and fixed solutions is less than 10 cm, in any component of the baseline (X,Y,or Z).

In the GPS data processing all the solutions from the bench marks satisfied these conditions except station J 697 for which RMS was 0.068. The distance from NPS Building 224 to J 697 is 33 km. Trimble recommends a baseline length greater than 30 km when the fixed solution does not meet the above conditions. The float solution should be used provided the RMS of fit is better than 0.08. Δh of double difference fixed solutions were used for the bench marks in this study. The double difference fixed solutions are shown in Table 3.

Table 3. DOUBLE DIFFERENCE FIXED SOLUTIONS

Bench mark	Slope distance from Bldg. 224 (m)	RMS (cycles)	Ratio	Coordinates difference between fixed and float solution ΔX (m)	ΔY (m)	ΔZ (m)
TREE	66	0.039	20.6	-2.0	0.3	-1.4
GH1	98	0.014	257.3	-0.5	0.4	0.0
GH2	256	0.025	94.7	-3.2	3.7	0.3
GH3	536	0.014	276.4	-1.1	0.6	-0.1
GH4	547	0.019	151.0	-0.7	0.4	-0.2
GH5	424	0.014	245.7	0.0	-1.6	-1.2
GH6	166	0.015	254.9	-0.7	0.5	-0.1
GH7	169	0.015	288.6	-0.7	0.3	-0.1
GH8	413	0.016	160.0	-0.4	0.5	0.1
K 152	14914	0.035	9.7	-0.3	0.1	-0.4
B 21	2082	0.014	193.5	0.4	-0.2	0.1
S 812	17062	0.020	15.8	-1.2	1.7	0.1
P 812	20011	0.020	31.7	0.9	-1.4	0.4
D 697	25249	0.045	11.1	8.2	9.3	0.4
J 697	32692	0.068	6.3	13.8	-10.7	2.6
F 813	16861	0.022	35.9	7.1	-5.2	0.1
GWM 27	16687	0.049	15.3	4.4	-5.7	0.5

Default parameters are assumed for automatic processing. These parameters have a minimum elevation mask of 15° for double differences. Station one coordinates were the best code positions during data logging. Actual surface meteorological values were used in the processing.

For sets of data covering more than one hour, epoch increments of five or ten provide full precision. Automatic processing begins with several iterations of triple differences to establish a starting value for the double difference solution. A cycle slip fixer processes every epoch.

Double difference solutions begin with several iterations of the float solution where biases, as well as baseline components, are computed. It is called the float solution, since the biases are allowed to float and are not constrained to be integers. Each iteration should indicate convergence toward zero for observed minus computed phases. The default elevation mask is 15° for

processing double differences. After completion of the float solution, correlations and biases are computed. The integer search algorithm begins to determine the proper integer values for the bias. After the biases are set to their integer values, the processing is repeated to compute the fixed solution. Ellipsoid height differences Δh for the temporary bench marks with respect to NPS Building 224, found using double difference fixed solutions is given in Table 4.

Table 4. Δh AND WGS 84 COORDINATES FOR THE TEMPORARY BENCH MARKS.

Bench mark	Δh (m)	X (m)	Y (m)	Z (m)
TREE	-8.3532	-2707298.015	-4353450.286	3781781.432
GH1	-9.1814	-2707385.942	-4353407.659	3781790.139
GH2	-3.7766	-2707502.139	-4353548.202	3781549.491
GH3	-0.6382	-2707528.391	-4353750.407	3781314.047
GH4	0.9055	-2707367.536	-4353813.483	3781306.605
GH5	-0.1099	-2707202.581	-4353799.531	3781493.061
GH6	-4.3397	-2707252.929	-4353587.524	3781666.956
GH7	-3.8509	-2707406.088	-4353553.901	3781589.428
GH8	0.0843	-2707329.452	-4353759.387	3781428.906

Δh for the permanent bench marks with respect to NPS Building 224 using double difference fixed solutions are given in Table 5.

Table 5. Δh AND WGS 84 COORDINATES FOR THE PERMANENT BENCH MARKS

Bench mark	Δh (m)	X (m)	Y (m)	Z (m)
K 152	2.2398	-2696890.306	-4350931.132	3792065.393
B21	-8.4275	-2708436.436	-4351973.362	3782674.213
S 812	1.6335	-2692137.111	-4360870.528	3784094.277
P 812	-1.3569	-2689245.909	-4360679.825	3786312.651
D 697	15.1728	-2685153.665	-4357222.820	3793175.171
J 697	71.3023	-2679276.491	-4356508.580	3798216.777
F 813	-6.7070	-2695613.779	-4350454.843	3793463.222
GWM 27	49.1051	-2692408.721	-4360427.654	3784433.846

2. Obtaining Geoid Height

Geoid heights of bench marks were obtained by using the Geoid.exe program from NGS or Trimvec (Table 6).

Table 6. GEOID HEIGHTS

Bench mark	NGS (m)	Trimvec (m)
TREE	-34.7	-34.30428
GH1	-34.7	-34.30671
GH2	-34.7	-34.31508
GH3	-34.7	-34.32072
GH4	-34.7	-34.31605
GH5	-34.7	-34.30745
GH6	-34.7	-34.30532
GH7	-34.7	-34.31140
GH8	-34.7	-34.31247
K 152	-34.1	-33.83303
B 21	-34.7	-34.31918
S 812	-33.9	-33.86473
P 812	-33.8	-33.76630
D 697	-33.6	-33.58265
J 697	-33.3	-33.42047
F 813	-34.0	-33.77899
GWM 27	-33.9	-33.86547

3. Determination of Local Geoid Model

The method of collocation was used to solve the local geoid model. The procedure is described as follows :

a. Reference Station Selection

Station TREE was selected as a reference station on the NPS campus. Coordinate differences ΔX, ΔY and ΔZ in WGS 84, for the temporary bench mark with respect to the TREE were computed. These are shown in Table 7.

Table 7. ΔX, ΔY AND ΔZ ON THE NPS CAMPUS

Bench mark	ΔX (m)	ΔY (m)	ΔZ (m)
TREE	0.000	0.000	0.000
GH1	-87.927	42.627	8.707
GH2	-204.124	-97.916	-231.941
GH3	-230.376	-300.121	-467.385
GH4	-69.521	-363.197	-474.827
GH5	95.434	-349.245	-288.371
GH6	45.085	-137.238	-114.476
GH7	-108.073	-103.615	-192.004
GH8	-31.437	-309.101	-352.526

Station K 152 was selected as a reference station in the large off campus area. The coordinate differences, ΔX, ΔY and ΔZ between the permanent bench marks and K 152 were computed in WGS 84 (Table 8).

Table 8. ΔX, ΔY AND ΔZ IN THE OFF CAMPUS AREA

Bench mark	ΔX (m)	ΔY (m)	ΔZ (m)
K 152	0.000	0.000	0.000
B 21	-11546.130	-1042.230	-9391.180
S 812	4753.195	-9939.396	-7971.116
P 812	7644.397	-9748.693	-5752.742
D 697	11736.641	-6291.688	1109.778
J 697	17613.815	-5577.448	6151.384
F 813	1276.527	476.289	1397.829
GWM 27	4481.585	-9496.522	-7631.547

b. Determine the Local Geoid Model

I start with assuming the signal, S, equals zero and the weight matrix **P**, equals the unit matrix. Seven control marks in both areas led to seven observation equations. The coordinate differences between the reference marks, which are TREE on the NPS campus and K 152 in the off campus area, are the coefficients of the parameters. The observation equation is given by

$$\Delta N = H + N_0(X,Y,Z) - \Delta h$$

where

H : the orthometric height, from levelling

$N_0(X,Y,Z)$: the global geoid height, obtained form NGS or Trimvec

Δh : the ellipsoid height difference with respect to the reference station

The local variation of the geoid height, ΔN, is also given by

$$\Delta N = h_0 + \text{combination of coordinate differences}$$

h_0, the ellipsoid height, includes a constant correction to $N_0(X,Y,Z)$ at the reference station. ΔN for temporary bench marks, $N_0(X,Y,Z)$ from the NGS (designated as NΔN) and $N_0(X,Y,Z)$ from the Trimvec (designated as TΔN) are given in Table 9.

Table 9. ΔN ON THE NPS CAMPUS

Bench mark	NΔN (m)	TΔN (m)
TREE	-26.3468	-25.9511
GH1	-26.3266	-25.9333
GH2	-26.3444	-25.9595
GH3	-26.3338	-25.9545
GH4	-26.3435	-25.9595
GH5	-26.3441	-25.9515
GH6	-26.3283	-25.9336

ΔN for permanent bench marks with $N_0(X,Y,Z)$ from the NGS (designated as NΔN) and $N_0(X,Y,Z)$ from the Trimvec (designated as TΔN) are given in Table 10.

Table 10. ΔN IN THE OFF CAMPUS AREA

Bench mark	NΔN (m)	TΔN (m)
K 152	-19.3748	-19.1078
B 21	-20.4855	-20.1047
S 812	-19.5555	-19.5202
P 812	-19.3091	-19.2754
D 697	-18.7158	-18.6984
J 697	-19.5413	-19.6618
F 813	-19.3100	-19.0890

A least squares adjustment program Lobs.basic was used to determine the geoid model which has the smallest standard deviation. This was done by using the different combinations of the coordinate differences. GPS data from the off campus area were studied to determine the local geoid model. $N_0(X,Y,Z)$ from the Trimvec program were used to compute ΔN.

Four-parameter combinations and their standard deviations for the off campus area are shown in Table 11.

Table 11. FOUR PARAMETERS

Parameters	Standard deviation (m)
$h_0 + a\Delta X + b\Delta Y + c\Delta Z$	0.45
$h_0 + a\Delta X + b\Delta Y + c\Delta Y^2$	0.52
$h_0 + a\Delta X + b\Delta Y + c\Delta X^2$	0.34
$h_0 + a\Delta X + b\Delta Y + c\Delta X\Delta Y$	0.47
$h_0 + a\Delta X + b\Delta Z + c\Delta Y^2$	0.52
$h_0 + a\Delta X + b\Delta Z + c\Delta X^2$	0.34
$h_0 + a\Delta X + b\Delta Z + c\Delta X\Delta Y$	0.47
$h_0 + a\Delta X + b\Delta Y^2 + c\Delta X^2$	0.31
$h_0 + a\Delta X + b\Delta X^2 + c\Delta X\Delta Y$	0.37
$h_0 + a\Delta Y + b\Delta Z + c\Delta Y^2$	0.52
$h_0 + a\Delta Y + b\Delta Z + c\Delta X^2$	0.34
$h_0 + a\Delta Y + b\Delta Z + c\Delta X\Delta Y$	0.46
$h_0 + a\Delta Y + b\Delta Y^2 + c\Delta X^2$	0.43
$h_0 + a\Delta Y + b\Delta Y^2 + c\Delta X\Delta Y$	0.58
$h_0 + a\Delta Y + b\Delta X^2 + c\Delta X\Delta Y$	0.28
$h_0 + a\Delta Z + b\Delta Y^2 + c\Delta X^2$	0.37
$h_0 + a\Delta Z + b\Delta X^2 + c\Delta X\Delta Y$	0.32
$h_0 + a\Delta Z + b\Delta Y^2 + c\Delta X\Delta Y$	0.49
$h_0 + a\Delta Y^2 + b\Delta X^2 + c\Delta X\Delta Y$	0.20 *

* The smallest standard deviation.

Five-parameter combinations and their standard deviations for the off campus area are given in Table 12.

Table 12. FIVE PARAMETERS

Parameters	Standard deviation (m)
$h_0 + a\Delta X + b\Delta Y + c\Delta Z + d\Delta X^2$	0.43
$h_0 + a\Delta X + b\Delta Y + c\Delta Z + d\Delta Y^2$	0.57
$h_0 + a\Delta X + b\Delta Y + c\Delta Z + d\Delta X\Delta Y$	0.39
$h_0 + a\Delta X + b\Delta Z + c\Delta X^2 + d\Delta Y^2$	0.57
$h_0 + a\Delta X + b\Delta Z + c\Delta Y^2 + d\Delta X\Delta Y$	0.59
$h_0 + a\Delta X + b\Delta X^2 + c\Delta Y^2 + d\Delta X\Delta Y$	0.22
$h_0 + a\Delta X + b\Delta Z + c\Delta X^2 + d\Delta Y^2$	0.57
$h_0 + a\Delta Y + b\Delta Z + c\Delta X^2 + d\Delta X\Delta Y$	0.34
$h_0 + a\Delta Y + b\Delta Z + c\Delta X^2 + d\Delta X\Delta Y$	0.34
$h_0 + a\Delta Y + b\Delta Z + c\Delta X^2 + d\Delta Y^2$	0.56
$h_0 + a\Delta Y + b\Delta X^2 + c\Delta Y^2 + d\Delta X\Delta Y$	0.10 *
$h_0 + a\Delta Z + b\Delta X^2 + c\Delta Y^2 + d\Delta X\Delta Y$	0.19

* The smallest standard deviation.

Six-parameter combinations and their standard deviations for the off campus area are given in Table 13.

Table 13. SIX PARAMETERS

Parameters	Standard deviation (m)
$h_0 + a\Delta X + b\Delta Y + c\Delta Z + d\Delta X^2 + e\Delta Y^2$	0.26
$h_0 + a\Delta X + b\Delta Y + c\Delta Z + d\Delta X^2 + e\Delta X\Delta Y$	0.59
$h_0 + a\Delta X + b\Delta Y + c\Delta X^2 + d\Delta Y^2 + e\Delta X\Delta Y$	0.13
$h_0 + a\Delta X + b\Delta Z + c\Delta X^2 + d\Delta Y^2 + e\Delta X\Delta Y$	0.12 *
$h_0 + a\Delta Y + b\Delta Z + c\Delta X^2 + d\Delta Y^2 + e\Delta X\Delta Y$	0.14

* The smallest standard deviation.

For seven observation equations and seven parameters the degree of freedom equals zero, so the standard deviation is undefined. Seven-parameter combinations and their standard deviations for the off campus area are given in Table 14.

Table 14. SEVEN PARAMETERS

Parameters	Standard deviation (m)
$h_0 + a\Delta X + b\Delta Y + c\Delta Z + d\Delta X^2 + e\Delta Y^2 + f\Delta X\Delta Y$	undefined

The best standard error of the four-parameter geoid model is greater than for five-parameter or six-parameter geoid models. The seven-parameter geoid model did not converge, so five-parameter and six-parameter geoid models were chosen for this study. The five-parameter geoid model is given by

$$N = N_0(X,Y,Z) + h_0 + a\Delta Y + b\Delta X^2 + c\Delta Y^2 + d\Delta X\Delta Y$$

and the six-parameter geoid height is given by

$$N = N_0(X,Y,Z) + h_0 + a'\Delta X + b'\Delta Z + c'\Delta X^2 + d'\Delta Y^2 + e'\Delta X\Delta Y$$

c. Evaluating the Geoid Model Using Check Marks

In order to evaluate the accuracy of the geoid model, the geoid height model can be rearranged to calculate the orthometric height of check marks which are GH7 and GH8 on the NPS campus and GWM 27 in the off campus area.

$$H = h_0 + \Delta h - N_0(X,Y,Z) + a\Delta Y + b\Delta X^2 + c\Delta Y^2 + d\Delta X\Delta Y$$

or

$$H = h_0 + \Delta h - N_0(X,Y,Z) + a'\Delta X + b'\Delta Z + c'\Delta X^2 + d'\Delta Y^2 + e'\Delta X\Delta Y$$

The h_0, a, b, c and d of the five-parameter geoid model, and h_0, a', b', c', d' and e' of the six-parameter geoid model, can be solved by least squares adjustments. The results for h_0, a, b, c and d for the five-parameter model using the seven control marks in the off campus area are given in Table 15. The results for h_0, a, b, c and d are shown in the NGS column, where the $N_0(X,Y,Z)$ was from the

NGS Geoid.exe program. The results for h_0, a, b, c, d and standard deviation, σ, are shown in the Trimvec column, where the $N_0(X,Y,Z)$ was from the Trimvec Geoid.exe program.

Table 15. h_0, a, b, c, d, σ FOR SEVEN CONTROL MARKS

Parameter	NGS (m)	Trimvec (m)
h_0	-19.26895	-19.00915
a	-2.344959E-04	-2.736598E-04
b	-8.523017E-09	-8.528133E-09
c	-3.574314E-08	-3.905052E-08
d	-2.211550E-08	-1.728313E-08
σ	0.130518	0.095723

The results of h_0, a, b, c, d and σ of the five-parameter geoid model using the six control marks (excluding B 21) in the off campus area are given in Table 16.

Table 16. h_0, a, b, c, d, σ FOR SIX CONTROL MARKS

Parameter	NGS (m)	Trimvec (m)
h_0	-19.25415	-19.01411
a	-2.752543E-04	-2.600850E-04
b	-7.735253E-09	-8.792540E-09
c	-3.777950E-08	-3.836067E-08
d	-1.798617E-08	-1.867193E-08
σ	0.171919	0.133516

The results of h_0, a, b, c, d and σ of the five-parameter geoid model using the six control marks (excluding J 697) in the off campus area are given in Table 17.

Table 17. h_0, a, b, c, d, σ FOR SIX CONTROL MARKS

Parameter	NGS (m)	Trimvec (m)
h_0	-19.265355	-19.03487
a	-2.503395E-04	-1.977533E-04
b	-8.731945E-09	-7.526410E-09
c	-3.711466E-08	-3.246532E-08
d	-2.179058E-08	-1.882791E-08
σ	0.1841317	0.1206984

The results of h_0, a, b, c and d of the five-parameter geoid model using the five control marks (excluding B 21 and J 697) in the off campus area are given in Table 18. The standard deviation is undefined since degree of freedom equals zero.

Table 18. h_0, a, b, c, d FOR FIVE CONTROL MARKS

Parameter	NGS (m)	Trimvec (m)
h_0	-19.37497	-19.10791
a	1.032352E-04	3.397465E-05
b	8.119969E-09	3.521564E-09
c	7.377821E-09	-3.339665E-09
d	1.338776E-09	-3.667083E-09

The results of h_0, a, b, c, d and σ of the five-parameter geoid model using the seven control marks on the NPS campus are given in Table 19.

Table 19. h_0, a, b, c, d, σ FOR SEVEN CONTROL MARKS

Parameter	NGS (m)	Trimvec (m)
h_0	-26.33543	-25.94105
a	2.976507E-06	1.192093E-07
b	-5.419133E-08	-1.763692E-07
c	-3.601599E-08	-8.489588E-08
d	6.309711E-08	-3.702007E-08
σ	0.013773	0.014612

The results of h_0, a', b', c', d', e' and σ of the six-parameter geoid model using the seven control marks in the off campus area are shown in Table 20.

Table 20. h_0, a', b', c', d', e', σ FOR SEVEN CONTROL MARKS

Parameter	NGS (m)	Trimvec (m)
h_0	-19.27388	-19.01660
a'	2.083182E-04	1.842380E-04
b'	-2.561510E-04	-2.520234E-04
c'	-7.731387E-09	-8.335974E-09
d'	-3.767036E-08	-3.960304E-08
e'	-1.249646E-08	-1.521767E-08
σ	0.137149	0.123960

The results of h_0, a', b', c', d' and e' of the six-parameter geoid model using the six control marks (excluding B 21) in the off campus area are given in Table 21. The standard deviation is undefined since degree of freedom equals zero.

Table 21. h_0, a', b', c', d', e' FOR SIX CONTROL MARKS

Parameter	NGS (m)	Trimvec (m)
h_0	-19.37489	-19.10785
a'	2.576411E-04	2.288222E-04
b'	-1.691282E-04	-1.734756E-04
c'	-1.083072E-08	-1.113654E-08
d'	-2.817797E-08	-3.102741E-08
e'	-5.824404E-09	-9.185896E-09

The results of h_0, a', b', c', d' and e' of the six-parameter geoid model using the six control marks (excluding J 697) in the off campus area are given in Table 22. The standard deviation is undefined since degree of freedom equals zero.

Table 22. h_0, a', b', c', d', e' FOR SIX CONTROL MARKS

Parameter	NGS (m)	Trimvec (m)
h_0	-19.37466	-19.10774
a'	1.065433E-04	9.238720E-05
b'	-4.604459E-05	-6.231666E-05
c'	-2.012712E-09	-3.170499E-09
d'	-1.141234E-08	-1.586523E-08
e'	-2.528395E-09	-6.206392E-09

The results of h_0, a', b', c', d', e' and σ of the six-parameter geoid model using the seven control marks on the NPS campus are given in Table 23.

Table 23. h_0, a', b', c', d', e', σ FOR SEVEN CONTROL MARKS

Parameter	NGS (m)	Trimvec (m)
h_0	-26.33389	-25.93820
a'	1.645088E-05	4.556775E-05
b'	4.225970E-05	6.169081E-05
c'	6.856863E-08	6.391201E-08
d'	6.007031E-08	5.878974E-08
e'	1.781154E-07	1.767185E-07
σ	0.018859	0.018872

Fortran program GPSDIS (Appendix C) was used to calculate the orthometric heights of the check marks. The results are discussed in the Chapter VII.

d. Obtaining the Weight Matrix

Since the standard error of the geoid model is about ± 2 cm, which is the same or better than GPS Δh accuracy, it is not actually necessary to do further computation. Nevertheless, the weight matrices, P, were computed.

The five-parameter geoid model used for the NPS campus and the six-parameter geoid model used for the off campus area were employed in obtaining P. The covariance function, C(r), of the control marks (p. 18) can be obtained by using the least squares residuals. The coefficients B and k of C(r) can be obtained by solving simultaneously the equations for C(r) with different distances from the centers.

(1) P for NPS Campus. The residuals of the five-parameter geoid model of the control marks on the NPS campus are given in Table 24. The global geoid heights were from the NGS.

Table 24. RESIDUALS FOR FIVE-PARAMETER MODEL

Control mark	# of residual	Residual (m)
TREE	V_1	1.137352E-02
GH1	V_2	-9.420395E-03
GH2	V_3	7.339478E-03
GH3	V_4	-4.278183E-03
GH4	V_5	3.572464E-03
GH5	V_6	6.446839E-04
GH6	V_7	-8.714676E-03

For $r_1 = 593$ m

$$C(r_1) = \frac{V_1V_4 + V_1V_5 + V_2V_4 + V_2V_5 + V_4V_6}{4} = -2.194272\text{E-}03$$

For $r_2 = 319$ m

$$C(r_2) = \frac{V_1V_3+V_2V_3+V_2V_7+V_3V_4+V_3V_5+V_3V_6+V_3V_7+V_4V_6+V_5V_6}{8} = 3.945813\text{E-}6$$

Thus the equations of covariance can be written as

$$C(r_1) = 0.0138 + B\sin(kr_1)$$

and

$$C(r_2) = 0.0138 + B\sin(kr_2)$$

Approximate solutions for coefficients B and k can be found by using the Maclaurin series expansion. The solved covariance function is given by

$$C(r) = 0.0138 - 0.0160\sin(1.7670\,r)$$

C_q can be obtained by using the C(r) with the distances between the control marks. Fortran program DISTCO (Appendix D) was used to compute the distances between the control marks, as well as C_q and, hence, the weight matrix, $P = C_q^{-1}$. The Matlab program on the NPS mainframe computer was used to find the inverse of C_q ($P = C_q^{-1}$). P for the five-parameter geoid model for the NPS campus is given by

$$\mathbf{P} = \begin{pmatrix} 162 & & & & & \text{ZEROS} & \\ & 142 & & & & & \\ & & 78 & & & & \\ & & & 98 & & & \\ & & & & 102 & & \\ & & & & & 87 & \\ & \text{ZEROS} & & & & & 100 \end{pmatrix}$$

(2) <u>P for off Campus Area</u>. The residuals of the six-parameter geoid model of the control marks in the off campus area are given in Table 25. The global geoid heights were calculated using the Trimvec program.

Table 25. RESIDUALS FOR SIX-PARAMETER MODEL

Control mark	# of residual	Residual (m)
K 152	V_1	9.120178E-02
B 21	V_2	-9.773254E-03
S 812	V_3	6.391526E-03
P 812	V_4	1.983643E-04
D 697	V_5	-2.779961E-02
J 697	V_6	1.685524E-02
F 813	V_7	-7.651711E-02

For r_1 = 13994 m

$$C(r_1) = \frac{V_1V_2 + V_1V_3 + V_1V_4 + V_1V_5 + V_3V_7 + V_4V_7}{5} = -6.659883E-04$$

For $r_2 = 19230$ m

$$C(r_2) = \frac{V_1V_6+V_2V_3+V_3V_6}{2} = 7.912463E\text{-}04$$

Thus the equations of covariance can be written as

$$C(r_1) = 0.0124 + B \sin(kr_1)$$

and

$$C(r_2) = 0.0124 + B \sin(kr_2)$$

Approximate solutions for coefficients B and k can be found by using a Maclaurin series expansion. The solved covariance function is given by

$$C(r) = 0.0124 - 0.1329 \sin(2.8278\ r)$$

Similarly, C_q can be obtained by using the C(r) with the distances between the control marks. Fortran program DISTCO (Appendix D) was used to compute the distances between the control marks and C_q, to obtain the weight matrix ($P = C_q^{-1}$). The Matlab program on the NPS mainframe computer was used to find the inverse of C_q ($P = C_q^{-1}$). P for the six-parameter geoid model of the off campus area is given by

$$\mathbf{P} = \begin{pmatrix} 25 & & & & & & \text{ZEROS} \\ & 15 & & & & & \\ & & 15 & & & & \\ & & & 16 & & & \\ & & & & 10 & & \\ & \text{ZEROS} & & & & 17 & \\ & & & & & & 25 \end{pmatrix}$$

VI. EVALUATION OF GPS RECEIVER ERRORS

Factors affecting the GPS measured ellipsoid height differences, Δh, include: (1) comparing GPS differencing solutions, (2) standard error of GPS observations, (3) corrections for surface meteorological values, and (4) observation durations for GPS.

A. COMPARING GPS DIFFERENCING SOLUTIONS

1. Ellipsoid Height Differences Tested at a Fixed Position

In order to determine the best solution for Δh using GPS, the Δh data from a Trimble 4000SX GPS receiver were collected in the K parking lot on the NPS campus from 0633 to 1007 UTC on February 19, 1988. The design of the experiments is shown in the Figure 10.

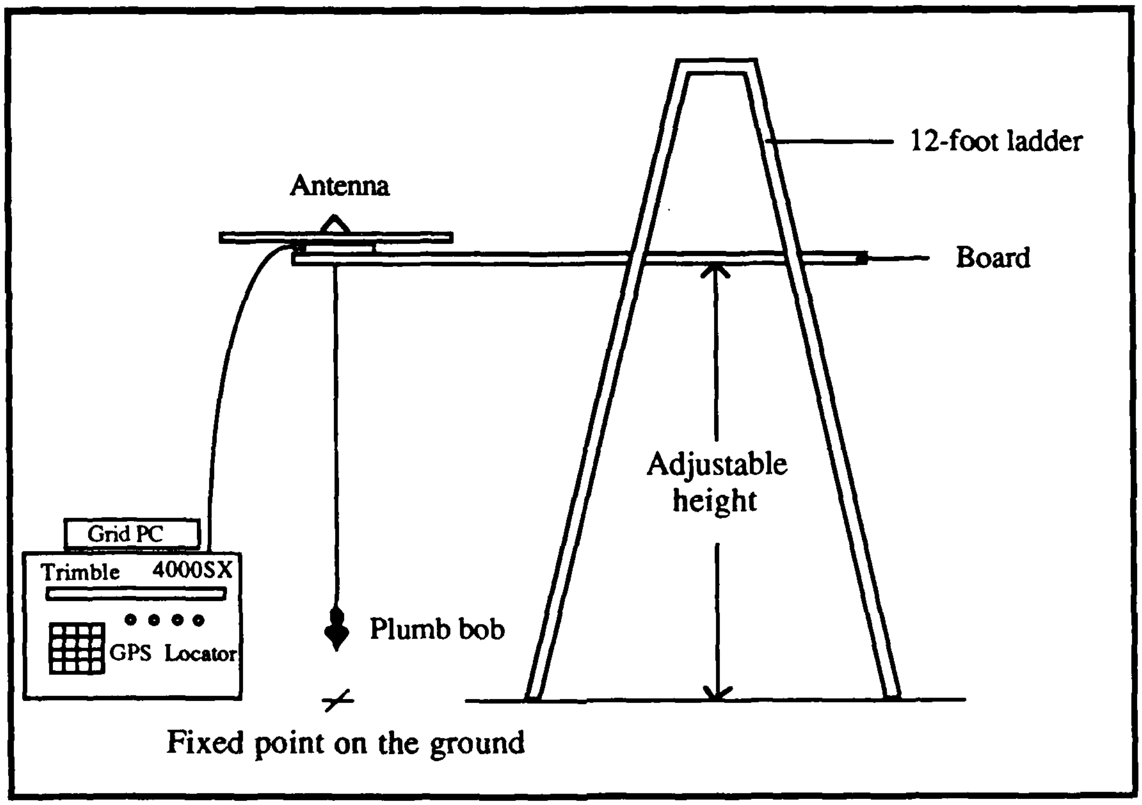

Figure 10. Illustration of the Δh test of Trimble 4000SX GPS receiver.

A plumb bob was used to point to a fixed position (36° 35' 56" N, 121° 52' 36" W) on the ground. The observations were taken by changing the antenna height over a distance of about 60 cm, which was measured with a tape every half hour. The ellipsoid height differences, Δh, and the orthometric height differences, ΔH, should agree with the change of antenna height for a fixed position. Triple difference solutions, Δh_{TRI}, are given in Table 26. Table also gives the differences between the Δh_{TRI} at succeeding observation times, $D\Delta h_{TRI}$; double difference float solutions, Δh_{FLT}; the differences between the Δh_{FLT} at succeeding observation times, $D\Delta h_{FLT}$; and double difference fixed solutions, Δh_{FIX}, and the differences between the Δh_{FIX} at succeeding observation times, $D\Delta h_{FIX}$.

Table 26. Δh AND DΔh FOR DIFFERENT ANTENNA HEIGHTS

TIME (UTC) (19-2-1988)	Δh_{TRI} (m)	$D\Delta h_{TRI}$ (m)	Δh_{FLT} (m)	$D\Delta h_{FLT}$ (m)	Δh_{FIX} (m)	$D\Delta h_{FIX}$ (m)
0633 - 0712	4.4410		4.4429		4.5045	
		-0.7189		-0.6339		-0.5463
0736 - 0808	5.1599		5.0768		5.0508	
		-0.3973		-0.5676		-0.5964
0824 - 0855	5.5572		5.6444		5.6472	
		-0.1954		-0.7144		-0.7030
0911 - 0943	5.7526		6.3588		6.3502	
		-0.4531		-0.2920		-0.2660
0945 - 1007	6.2057		6.6508		6.6162	

The differences between the GPS $D\Delta h$ and the differences of the measured ellipsoid height differences, $D\Delta h_{MEA}$, are given in Table 27.

Table 27. **THE DIFFERENCE OF Δh**

ANTENNA HEIGHT (m)	$D\Delta h_{MEA}$ (m)	$D\Delta h_{MEA} - D\Delta h_{TRI}$ (m)	$D\Delta h_{MEA} - D\Delta h_{FLT}$ (m)	$D\Delta h_{MEA} - D\Delta h_{FIX}$ (m)
2.809				
	-0.5470	0.1719	0.0869	-0.0007
2.262				
	-0.5960	-0.1987	-0.0284	0.0004
1.666				
	-0.5920	-0.3966	0.1224	0.1110
1.074				
	-0.5900	-0.1369	-0.2980	-0.3240
0.484				

The results show that the double difference fixed solution is the best. The difference between the measured ellipsoid height difference and the GPS ellipsoid height difference was about 1 mm. The last two results were not good because they had multipath effects and imaging effects from the ground (antenna heights 1.074 m and 0.484 m), so the solutions of ΔH compared to the true differences were large. This indicates that the antenna should be at least 1 m above the ground.

2. <u>Ellipsoid Height Differences Tested at Different Positions</u>

Since the geoid on the NPS campus has a flat geoid slope, the differences between Δh, $D\Delta h$, and the orthometric height difference, ΔH, of the temporary bench marks can be neglected. The Δh and $D\Delta h$ from the GPS observations are given in Table 28. The differences between the GPS $D\Delta h$ and the ΔH are given in Table 29.

Table 28. Δh AND $D\Delta h$ ON THE NPS CAMPUS

Bench mark	Δh_{TRI} (m)	$D\Delta h_{TRI}$ (m)	Δh_{FLT} (m)	$D\Delta h_{FLT}$ (m)	Δh_{FIX} (m)	$D\Delta h_{FIX}$ (m)
TREE	-8.3358		-8.3375		-8.3532	
		0.8501		0.8313		0.8282
GH1	-9.1859		-9.1688		-9.1814	
		-5.3472		-5.3690		-5.4048
GH2	-3.8387		-3.7998		-3.7766	
		-3.1994		-3.1624		-3.1384
GH3	-0.6393		-0.6374		-0.6382	
		-1.5241		-1.5441		-1.5437
GH4	0.8848		0.9067		0.9055	
		0.9777		1.0171		1.0154
GH5	-0.0929		-0.1104		-0.1099	
		4.2655		4.2303		4.2298
GH6	-4.3584		-4.3407		-4.3397	
		-0.4932		-0.4916		-0.4888
GH7	-3.8652		-3.8491		-3.8509	
		-3.9412		-3.9360		-3.9352
GH8	0.0760		0.0869		0.0843	

Table 29. DIFFERENCES BETWEEN $D\Delta h$ AND ΔH

Bench mark	H (m)	ΔH (m)	$\Delta H - D\Delta h_{TRI}$ (m)	$\Delta H - D\Delta h_{FLT}$ (m)	$\Delta H - D\Delta h_{FIX}$ (m)
TREE	0.000				
		0.808	-0.0423	-0.0235	-0.0204
GH1	-0.808				
		-5.387	-0.0398	-0.0180	0.0178
GH2	4.579				
		-3.149	0.0504	0.0134	-0.0106
GH3	7.728				
		-1.534	-0.0099	0.0101	0.0097
GH4	9.262				
		1.016	0.0383	-0.0011	0.0006
GH5	8.246				
		4.214	-0.0515	-0.0163	-0.0158
GH6	4.032				
		-0.489	0.0042	0.0026	-0.0002
GH7	4.521				
		-3.924	0.0172	0.0120	0.0112
GH8	8.445				

It is clear that the double difference fixed solutions give the best solution in a flat geoid slope area.

B. STANDARD ERROR OF GPS OBSERVATIONS

To determine the accuracy of GPS, measurements were taken at bench mark TREE on the NPS campus. Three measurements, each of about three hours duration, were taken for this analysis. Double difference fixed solutions were used to analyze the data. The double difference fixed solutions for these measurements are given in Table 30.

Table 30. Δh_{FIX} AT STATION TREE

Date	Time (UTC)	Δh_{FIX} (m)
Feb. 5, 88	0730 - 1025	-8.4262
Feb. 7, 88	0721 - 1004	-8.3532
Feb. 11, 88	0705 - 0959	-8.3602

The standard error of the mean of Δh ($\sigma / \sqrt{n}$) is about $\pm$ 2 cm at TREE and the standard error (σ) of the single measurement is about $\pm$ 4 cm for three hours of observation.

Results obtained by Strange in 1985 for a project southeast of Phoenix, Arizona, show that using multiple GPS observations at the same point lead to uncertainties typically less than 2 cm in Z-direction over 20-km distances [Zilkoski, 1988].

Thus the accuracy of the ellipsoid differences obtainable by GPS measurements is expected to be about $\pm$ 2 cm for about three hours of observation.

C. CORRECTIONS FOR SURFACE METEOROLOGICAL VALUES

The meteorological correction is a function of the atmospheric refractivity as computed by the surface meteorological values of pressure, temperature, humidity and the elevation angle of the satellite. Larger corrections are required for low elevation angles, as the signal travels a longer path through the troposphere. A modified Hopfield troposphere model [Fell, 1975] is used for

automatic processing of Trim640 software. The model corrects for at least 90% of the tropospheric delay [Remondi, 1984]. Default surface meteorological values for automatic processing are 1010 millibars, 20° C and 50 % relative humidity. These parameters can be reset before the processing. Trim640 allows only one pressure, temperature and humidity entry for each site per session. Ellipsoid height differences, Δh_{FIX}, were calculated for four marks on the NPS campus and two off campus using the default meteorological parameters and using the true values. There are given in Table 31 along with differences found between the two methods.

Table 31. COMPARISON OF SURFACE METEOROLOGICAL CORRECTION

Bench mark	Default Δh_{FIX} (m)	True Δh_{FIX} (m)	Difference (mm)
GH2	-3.7777	-3.7766	1.1
GH3	-0.6376	-0.6383	0.6
GH4	0.9070	0.9055	1.5
GH7	-3.8524	-3.8509	1.5
K 152	2.2398	2.2430	-3.2
J 697	-71.3179	-71.3023	15.6

The difference is about ± 2 mm on the NPS campus and about ± 2 cm in the Monterey Bay area. The Trim640 program contains no ionospheric model. For first-order relative positioning (1 part in 10^5) and a baseline shorter than 50 km ionospheric delay can be neglected [Remondi, 1984]. In this study surface meteorological corrections were made during data processing to get the best Δh solutions.

D. OBSERVATION DURATIONS FOR GPS

To find out how long observations must be made to obtain the best solutions in the field when batteries are used for power, the GPS data from S 812, J 697 and K 152 were segmented into averaging periods of length $n\Delta t$, where $\Delta t = 10$ minutes and $n = 1, 2, 3, ...,10$. Default meteorological values were used for this processing. Δh_{FIX} for various observation durations for the stations is given in Table 32 and plotted in Figure 11 to Figure 13.

Table 32. Δh_{FIX} AS A FUNCTION OF OBSERVATION DURATION

Time (min)	S 812 (m)	J 697 (m)	K 152 (m)
10	1.5309	71.5291	2.3135
20	1.6141	70.9697	2.3102
30	1.6441	71.2185	2.3062
40	1.6539	71.4514	2.3065
50	1.6558	71.4762	2.3060
60	1.6498	71.5343	2.2563
70	1.6464	71.5309	2.2504
80	1.6349	71.3407	2.2504
90		71.3179	2.2548
100			2.2430

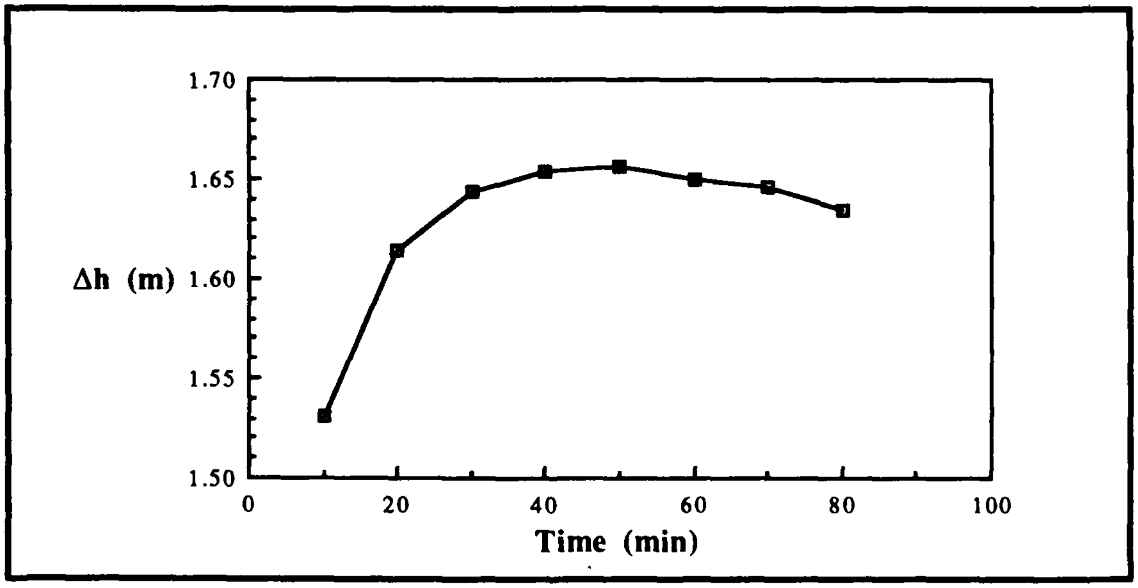

Figure 11. Δh vs. observation durations at station S 812.

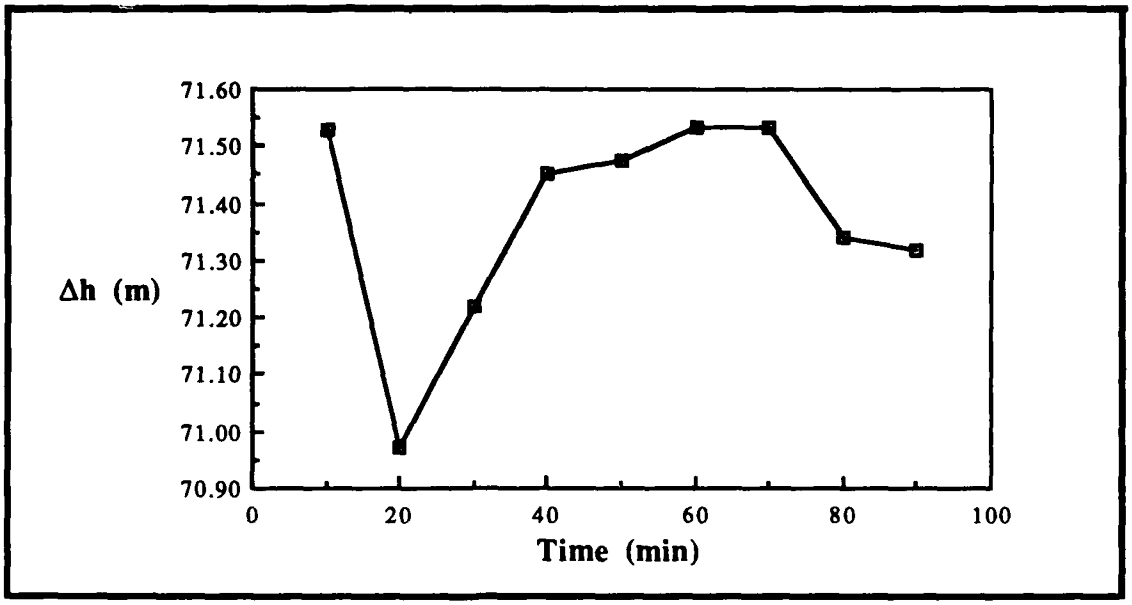

Figure 12. Δh vs. observation durations at station J 697.

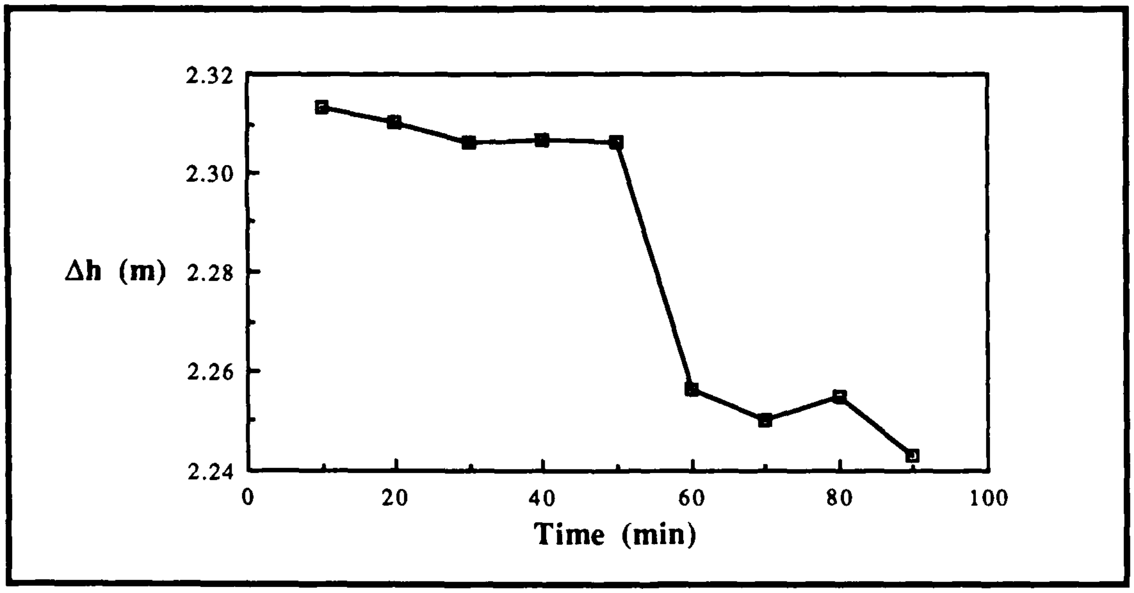

Figure 13. Δh vs. observation durations at station K 152.

It is clear that Δh was affected by the observation duration. Δh varied quite a lot in the first forty minutes. After about forty minutes, Δh tended to close to the mean value of the observations. After sixty minutes, Δh didn't vary much. Trimble recommends, to ensure sufficient quality data and to obtain first-order results on single observations, at least one hour should be taken for a four or five

satellite session. For baseline lengths up to 30 km the conditions for the fixed double solution of automatic processing will be met if data are collected for ninety minutes on four or five satellites [Trimble, 1987b]. The results shows that the best solutions can be improved with longer observation times. Normally, at least sixty minutes of observations are required for vertical control.

VII. RESULTS AND DISCUSSION

A. EVALUATATION OF GEOID MODEL

To find the accuracy of the geoid model the orthometric heights of the check marks from GPS, H_{GPS}, were calculated, and these were compared to the orthometric height of the check marks from the levelling, $H_{LEVELLING}$.

I started with the assumptions that $C_q = \mathbf{I}$ and $C_{pq} = 0$. Five-parameter and six-parameter geoid models were used to calculate the orthometric height of the check marks. GWM 27 is a check mark in the off campus area. GH7 and GH8 are the check marks on the NPS campus. $N_0(X,Y,Z)$ was calculated using both the NGS and the Trimvec programs.

1. H_{GPS} from Five-Parameter Geoid Model

a. H_{GPS} in the off Campus Area

H_{GPS} in GWM 27 was calculated by using seven control marks, six control marks (excluding B 21 or J 697) and five control marks (excluding B 21 and J 697). Comparisons of H in GWM 27 are given in Table 33.

Table 33. COMPARISONS OF H USING FIVE-PARAMETER MODEL AT GWM 27

GWM 27	$H_{GPS} - H_{LEVELLING}$ (cm)	
# of control marks	Trimvec	NGS
7	-11.55	-2.24
6 (excluding B21)	-9.77	-8.07
6 (excluding J 697)	-4.87	-3.65
5	2.14	6.58

b. H_{GPS} on the NPS Campus

H_{GPS} in GH7 and GH8 were calculated by using seven control marks. Comparisons of H in GH7 and GH8 are given in Table 34.

Table 34. COMPARISONS OF H USING FIVE-PARAMETER MODEL AT GH7 AND GH8

Five parameters	H_{GPS} - $H_{LEVELLING}$ (cm)	
Check mark	Trimvec	NGS
GH7	0.52	0.82
GH8	-0.20	0.00

2. H_{GPS} from Six-Parameter Geoid Model

a. H_{GPS} in the off Campus Area

H_{GPS} in GWM 27 were calculated by using seven control marks, and six control marks (excluding B 21 or J 697). Comparisons of H at GWM 27 are given in Table 35.

Table 35. COMPARISONS OF H USING SIX-PARAMETER MODEL AT GWM 27

GWM 27	H_{GPS} - $H_{LEVELLING}$ (cm)	
# of control marks	Trimvec	NGS
7	-12.40	-11.13
6 (excluding B21)	-9.37	-7.72
6 (excluding J 697)	-3.49	-1.01

b. H_{GPS} on the NPS Campus

H_{GPS} of GH7 and GH8 were calculated by using seven control marks. Comparisons of H at GH7 and GH8 are given in Table 36.

Table 36. COMPARISONS OF H USING SIX-PARAMETER MODEL AT GH7 AND GH8

Six parameters	H_{GPS} - $H_{LEVELLING}$ (cm)	
Check mark	Trimvec	NGS
GH7	1.22	1.22
GH8	0.22	0.25

Since the accuracy of predicted geoid height at check marks (± 0.2 to 2 cm) is less than the accuracy of GPS observations (± 2 to 4 cm), it may be

concluded that P = I is a satisfactory covariance matrix and further computation is unnecessary.

B. ACCURACY

The best accuracy of the five-parameter geoid model is ± 2 cm in the off campus area and is ± 2 to 10 mm for the NPS campus. The best accuracy of six-parameter geoid model is ± 1 cm in the off campus area and is ± 2 to 10 mm on the NPS campus.

C. DISCUSSION

The errors in the geoid model include errors associated with GPS and with levelling.

1. Errors Associated With GPS

a. Atmospheric Delays

The time delay of RF signals passing through the ionosphere is due to a reduction in speed and the bending of the ray, both effects being due to refraction. The overall delay in the signal is nearly inversely proportional to the square of the frequency. The ionospheric delay may be calculated by using two-frequency receivers (C/A code, P code), and corrected for with a satisfactory degree of accuracy. Tropospheric delays are independent of frequency. They are relatively small and can be modelled using the elevation angle of the satellite. The Trimble 4000SX receiver only observes the C/A code . The Trim640 program contains no ionospheric modelling. The modified Hopfield model [Fell, 1975] is used for the tropospheric delay. The combined effects of unmodelled ionospheric and tropospheric errors are estimated to result in a satellite-to-user range error of from 2.4 to 5.2 m [Milliken, 1980].

b. Selection of Appropriate Control Marks

Only eight permanent bench marks were recovered in the off campus study area. D 697 was obstructed by a tree. F 813 was set near a steel bridge. Although offset levelling was performed, the ellipsoid height could have been affected by offsetting. The shape of the bench marks were spread out roughly in a rhomboid shape. B 21 and J 697 are on the ends of the rhomboid. The distance is about 33 km from B 21 to J 697. The elevation changes form 5.787 to 85.061 m. If more bench marks could be recovered and be connected to these marks, the accuracy of the seven control marks could be improved.

c. Error of Antenna Height Measurements

The slant heights of GPS antenna were measured by using a steel tape before and after data collection. Thus, the heights of antenna were calculated. The systematic error of the tape, reading error, calculation error and a slack tape doing measurement will affect the accuracy of the ellipsoid height difference. The error of antenna height measurement is ± 2 to 10 mm.

d. Selecting observation period with optimal satellite geometry

Because there are only seven GPS satellites in operation at present, the observing window is only four to five hours a day for four or five satellites. The observation schedule should be planed early and considered the travel time, and time to reoccupy the marks. In this study most of the GPS measurements were made at night. PDOP were not always optimum.

e. Error in the Computing of Trim640 Program

There is no standard specification of the GPS software at present. Round off error in the data processing, the data record frequency, and the precision of the ephemeris could affect the components of the baseline. Specifications of the GPS software certainly will be developed in the future.

2. <u>Errors in Levelling</u>

The error in levelling would cause the errors of the observation equations when determining the local geoid model. The maximum allowable closing error between the forward and backward running of a section is 3.0 mm $\sqrt{k}$ for first-order class I levelling, and 9.0 mm $\sqrt{k}$ for third-order class I levelling, where k is the length of the section measured in km [Bodnar, 1975]. The average distance from K 152 to the permanent bench marks is about 13 km in the off campus study area. Thus, for first-order class I levelling, the maximum allowable closing error would be about 11 mm in the off campus area. The first-order bench marks were set around 1933. The elevations of the marks may have changed caused by subsidences or earthquakes. Since the time did not permit rerunning the levelling for the off campus area, published elevations were assumed correct. Errors in levelling could contribute to errors in the geoid model.

The distance of the interlocking levelling circuit on the NPS campus is about 0.8 km. For third-order class I levelling, the maximum allowable closing error is about 7 mm on the NPS campus. The 5 mm closing error was obtained

for that region. The geoid height model thus had a better accuracy on the NPS campus.

VIII. CONCLUSIONS AND RECOMMENDATIONS

Local geoid models for the Monterey, California, area were determined by GPS differential positioning using the method of collocation. The local geoid models were based on Rapp's 360 degree x 360 order global geoid models determined from gravity measurements. Control data were adjusted by least squares to solve for the parameters in the local geoid model. Also studied were factors that affected the GPS-measured ellipsoid height differences. These included (1) comparing GPS differencing solutions, (2) standard error of GPS observations, (3) corrections for surface meteorological values, and (4) observation durations for GPS. Conclusions are the following:

1. The accuracy of a local geoid height, N, is the same, whether the NGS or the Trimvec version of Rapp's global geoid model is used, although $N_0(X,Y,Z)$ calculated using Trimble's Trimvec program gives the best results for points on the NPS campus, and $N_0(X,Y,Z)$ from the NGS program gives the best results for the larger study area. For practical use the orthometric heights can be calculated by utilizing the local geoid model.

2. The areas where the geoid is smoothest lead to the best results. This was found by comparing the local geoid model used in the larger Monterey Bay area to the local geoid model used on the NPS campus. Also the density of control marks on the NPS campus was much higher than for the larger study area. If a higher accuracy is required in a large area where the geoid may be undulating, more control marks are required. These control marks should be strategically located throughout the network in order to determine the geoid slope.

3. The local geoid model does improve the accuracy of Rapp's global geoid model locally. The accuracy for Rapp's model is ± 2 m in an area 55 km x 55 km. The accuracy for the local geoid model is ± 2 cm in an area 30 km x 30 km and ± 1 cm in an area 0.5 km x 0.5 km.

4. A GPS double difference fixed solution gives the best ellipsoid height differences, Δh, for baseline lengths up to 30 km.

5. The standard error of GPS observations of the ellipsoid height difference, Δh, is ± 2 cm to ± 4 cm, so there is no need to perform the collocation with a non-unit weight matrix. Also it is not possible to reduce the error of the GPS observation below ± 2 cm.

6. There is no need for local surface meteorological corrections to the GPS observations. Default meteorological values (1010 millibars, 20° C and 50% relative humidity) are sufficient to meet the required accuracy.

7. The longer the duration of the GPS measurements, the better the result will be. The accuracy of the geoid model for the NPS campus, where GPS measurements were made over four- to five-hour periods, is better than the accuracy of the geoid height model in the larger study area, where measurements were made for periods of only ninety minutes.

8. A five-parameter geoid model gives the best solution for the NPS campus, while a six-parameter geoid model gives the best solution for the larger study area. The five-parameter geoid model which does not include a ΔZ term should be used with caution where ΔZ greatly exceeds 5 m.

Recommendations are as follows:

1. The GPS antenna should be set at least 1 m above the ground to reduce the multiple effects from the ground. The height of the antenna should be carefully measured before and after the data collection.

2. The configuration of the control marks should be carefully planned. Sites should be selected to optimize connections to stations with known precise orthometric height differences, minimizing the effects of obstructions. Control marks in a large area should be numerous enough to meet the required accuracy.

3. To minimize computation errors least squares adjustments should be made using double precision processing when solving the geoid model. For large amounts of GPS data a mainframe computer should be used rather than a microcomputer to reduce processing time.

APPENDIX A. FORTRAN PROGRAM GPSCON

```
*******************************************************************
*     WRITTEN BY MA, WEI-MING    05/08/88                         *
*     THIS PROGRAM READS DATA FROM GEOID DATA FILE AND CALLS       *
*     THE SUBROUTINE CONTOR TO PLOT GEOID HEIGHTS.                *
*     THE VARIABLES USED ARE:                                     *
*        GEOHT   :   GEOID HEIGHTS (m)                            *
*          NX    :   NUMBER OF POINTS IN THE X-DIRECTION          *
*          NY    :   NUMBER OF POINTS IN THE Y-DIRECTION          *
*                                                                 *
*******************************************************************
      REAL*4  GEOHT(22,15)
      DATA   NX,NY/22,15/
C
C READ GEOID HEIGHTS FROM GEOID1 DATA FILE
C
      CALL EXCMS('FILEDEF 8 DISK GEOID1 DATA A1')
      DO 200 J = 1, NY
         DO 100 I = NX, 1, -1
            READ(8,*) GEOHT(I,J)
 100     CONTINUE
 200  CONTINUE
C
C PLOT THE GEOID   .IGHT BY CALLING SUBROUTINE CONTOR
C
      CALL CONT  .R(GEOHT,NX,NY)
      STOP
      END
      SUBROUNTINE CONTOR(A,NX,NY)
*******************************************************************
*                                                                 *
*     WRITTEN BY MA, WEI-MING    05/08/88                         *
*     THIS SUBROUTINE CONTOURS AN NX BY NY ARRAY OF               *
*     REGULARLY SPACED POINTS.                                    *
*     NOTE: THIS ARRAY MUST BE REAL*4.                            *
*     THE VARIABLES USED ARE:                                     *
*           A    :   SINGLE PRECISION NX BY NY ARRAY OF           *
*                    REGULARLYSPACED POINTS.                      *
*          NX    :   NUMBER OF POINTS IN THE X-DIRECTION          *
*          NY    :   NUMBER OF POINTS IN THE Y-DIRECTION          *
*         ZINC   :   CONTOUR INTERVAL                             *
*                                                                 *
*******************************************************************
      DIMENSION A(NX,NY)
      COMMON WORK(5000)
C
C SET PARAMETERS FOR AXES
C
```

```
      XORIG = -55.0
      XSTP = 2.0
      XMAX = -34.0
      YORIG = 35.0
      YSTP = 2.0
      YMAX = 49.0
C
C SET CONTOUR LEVEL
C
      ZINC = 0.1
      CALL COMPRS
      CALL SETCLR('CYAN')
C
C SET PAGE AND PLOT SIZES, SET UP AXES FOR PLOT
C
      CALL PAGE(8.0,6.0)
      CALL BCOMON(5000)
      CALL AREA2D(7.0,5.0)
C
C LABEL AXES
C
      CALL XNAME('LONGITUDE 121 DEGREE WEST (MINUTES)$',100)
      CALL YNAME('LATITUDE 36 DEGREE NORTH (MINUTES)$',100)
      CALL GRAF(XORIG,XSTP,XMAX,YORIG,YSTP,YMAX)
      CALL FRAME
C
C MAKE CONTOURS AND DRAW
C
      CALL SETCLR('RED')
      CALL CONMIN(3.0)
      CALL CONANG(60.)
      CALL CONLIN(0,'MYCON','LABELS',1,10)
      CALL CONMAK(A,NX,NY,ZINC)
      CALL CONTUR(1,'LABELS','DRAW')
C
C END PLOT
      CALL ENDPL(0)
      CALL DONEPL
      RETURN
      END
C     SUBROUTINE MYCON(RARAY,IARAY)
**********************************************************************
*                                                                    *
*   THIS SUBROUTINE MAKES NEGATIVE CONTOURS DASHED AND               *
*   THE ZERO LINE HEAVYIER                                           *
*                                                                    *
**********************************************************************
      DIMENSION RARAY(2),IARAY(1)
      CALL RESET('DASH')
      IF (RARAY(1).GE. 0.) GO TO 10
      CALL DASH
   10 RARAY(2) = 1.
      IARAY(1) =1
```

```
IF (RARAY(1).EQ.0.) IARAY(1) = 2
RETURN
END
```

APPENDIX B. BASIC PROGRAM LOBS.BASIC

```
5      REM LEAST SQUARES BY OBSERVATION
10     DIM L(N%,1)
20     DIM A(N%,X%), P(X%,X%), EX(X%,1)
30     DIM CX(X%,1)
40     DIM AT(X%,N%), R(50), ATP(X%,X%)
50     DIM ATPA(X%,X%), ATPL(X%,1), AI(X%,X%)
69     PRINT " INPUT # OF OBSERVATIONS"
70     NPUT NO
200    PRINT " INPUT # OF PARAMETERS"
270    INPUT N1
350    ERASE L
351    ERASE A
352    ERASE AT
353    ERASE P
354    ERASE ATP
355    ERASE ATPL
356    ERASE AI
357    ERASE CX
358    ERASE EX
359    ERASE ATPA
380    DIM L(NO,1), A(NO,N1), AT(N1,NO), P(NO,NO), ATP(N1,NO),
ATPL(N1,1),AI(N1,N1)
390    DIM CX(N1,1), EX(NO,1), ATPA(N1,N1)
400    ITERO = 0
1180   PRINT " INPUT COEFFICIENTS OF OBSERVATION EQUATIONS AND
WEIGHTS"
1190   FOR K = 1 TO NO
1200   FOR J = 1 TO N1
1220   PRINT "COEFFICIENT ", K, J
1230   INPUT A(K,J)
1240   PRINT A(K,J)
1250   NEXT J
1260   PRINT " INPUT OBSERVED VALUE ", K
1270   INPUT L(K,1)
1280   PRINT " INPUT WEIGHT OF OBSERVED VALUE ", K
1300   INPUT P(K,K)
1390   NEXT K
1410   DOF = 0
1420   REM LEAST SQUARES
1430   M = NO
1450   L = N1
1460   PRINT " M =", M, "N1 = ", N1, "L = "L
1470   FOR I = 1 TO M
1480   FOR J = 1 TO L
1490   AT(J,I) = A(I,J)
1500   NEXT J
1510   NEXT I
1520   REM ATP = AT * P
1530   N = NO
1540   FOR I = 1 TO L
```

```
1550  FOR J =1  TO N
1560  ATP(I,J) = 0
1570    FOR K = 1  TO M
1580    ATP(I,J) = ATP(I,J) + AT(I,J) * P(K,J)
1590  NEXT K
1600  NEXT J
1610  NEXT I
1620  REM  ATPA = ATP * A
1630  N = N1
1640  FOR I = 1 TO L
1650  FOR J = 1 TO N
1660  ATPA(I,J) = 0
1680  ATPA(I.J) = 0
1690  FOR K = 1 TO  M
1710  ATPA(I,J) = ATPA(I,J) + ATP(I,K) * A(K,J)
1720  NEXT K
1730  NEXT J
1740  NEXT I
1750  REM  ATPL = ATP * L
1760  N = 1
1770  FOR I = 1 TO L
1780  FOR J = 1 TO N
1790    ATPL(I,J) = 0
1800    FOR K = 1  TO M
1810    ATPL(I,J) = ATPL(I,J) + ATP(I,K) * L(K,J)
1820  NEXT K
1830  NEXT J
1840  NEXT I
1970  REM  AI = INV(ATPA)
1980  I = N1
1990  M = N1
2000  N = I - 1
2020  MI = M - 1
2030  FOR J = 1 TO I
2040  FOR K = 1 TO I
2050  AI(J,K) = ATPA(J,K)
2055  NEXT K
2060  NEXT J
2070  FOR K =1 TO I
2080  FOR J = 1 TO MI
2090      R(J) = AI(1,J+1) / AI(1,1)
2100  NEXT J
2110      R(M) = 1! / AI(1,1)
2120  FOR L = 1 TO N
2130  FOR J  = 1 TO MI
2140      AI(I,J) = AI(L+1,J+1) - AI(L+1,1) * R(J)
2150  NEXT J
2160      AI(L,M) = -AI(L+1,1) * R(M)
2170  NEXT L
2180  FOR J = 1 TO M
2190      AI(I,J) = R(J)
2200  NEXT I
2210  NEXT K
```

```
2220 REM CX = AI * ATPL
2230 L = N1
2240 N = 1
2250 M = N1
2260 FOR I = 1 TO L
2270 FOR J = 1 TO N
2280 CX(I,J) = O
2300 FOR K = 1 TO M
2310 CX(I,J) = CX(I,J) + AI(I,K) * ATPL (K,J)
2320 NEXT K
2325 PRINT " PARAMETER # ", I, CX(I,J)
2330 NEXT J
2340 NEXT I
2480 PRINT " RESIDUAL"
2490 L = NO
2500 N = 1
2510 M = N1
2520 REM EX = A * CX
2530 FOR I = 1 TO L
2540 FOR J = 1 TO N
2560 EX(I,J) = 0
2570 FOR K = 1 TO M
2580 EX(I,J) = EX(I,J) + A(I,K) * CX(K,J)
2590 NEXT K
2600 NEXT J
2610 NEXT I
2620 SD = 0
2630 FOR K = 1 TO NO
2640 V = EX(K,1) - L(K,1)
2650 PRINT K, V
2660 SD = SD + V * V
2670 NEXT K
2680 SD = SQR(SD / (NO - N1 + DOF))
2690 PRINT SD
2695 PRINT " STD.DEV", SD
2700 ITER = ITER + 1
2710 IF ITER < 3 GOTO 350
2720 END
```

APPENDIX C. FORTRAN PROGRAM GPSDIS

```
**********************************************************************
*    WRITTEN BY WEI-MING MA  05/14/88                                 *
*    THIS PROGRAM READS DATA FROM TERMINAL  AND COMPUTES              *
*    THE ORTHOMETRIC  HEIGHTS FOR THE CHECK MARKS THEN                *
*    COMPUTES THE DIFFERENCE.                                         *
*    THE VARIABLES USED ARE:                                          *
*     RX,RY,RZ :  THE COORDINATES OF THE REFERENCE MARK (m)           *
*     CX,CY,CZ :  THE COORDINATES OF THE CHECK MARK (m)               *
*     DX,DY,DZ :  THE COORDINATES DIFFERENCE (m)                      *
*        H     :  THE ORTHOMETRIC HEIGHT OF CHECK MARK (m)            *
*        LH    :  THE LEVELED ORTHOMETRIC HEIGHT (m)                  *
*        DH    :  THE ELLIPSOID HEIGHT DIFFERENCE (m)                 *
*        GN    :  THE GLOBAL GEOID HEIGHT OF CHECK MARK (m)           *
*        LH    :  THE LEVELED ORTHOMETRIC HEIGHT (m)                  *
*        DIF   :  THE  DIFFERENCE BETWEEN H AND LH (cm)               *
*      A,B,C,D :  THE COEFFICIENTS OF 5-PARAMETER GEOID MODEL         *
*     A1,B1,C1 :  THE COEFFICIENTS OF 6-PARAMETER GEOID MODEL         *
*      D1,E1                                                          *
*     IPA,IPA1 :  THE NUMBER OF PARAMETERS                            *
*        IW    :  THE OUTPUT UNIT                                     *
*                                                                     *
**********************************************************************
      REAL*8      RX, RY, RZ, CX, CY, CZ, DX, DY, DZ, LH, GN,
      REAL*8      A, B, C, D, A1, B1, C1, D1, E1, H, DH, DIF,
      INTEGER IPA, IPA1, IW
      CHARACTER*1  RESPN1, RESPN2, RESPN3
      IW = 6
      IPA = 0
      IPA1 = 0
C
C READ DATA FROM TERMINAL
C
 10   PRINT *, 'ENTER # OF PARAMETERS'
      READ *, IPA
      IF (IPA .EQ. 5 .OR. IPA .EQ. 6) THEN
         PRINT *, 'ENTER THE X COORDINATE IN WGS 84 OF THE ',
     &            'REFERENCE MARK'
         READ *, RX
         PRINT *, 'ENTER THE Y COORDINATE IN WGS 84 OF THE ',
     &            'REFERENCE MARK'
         READ *, RY
         PRINT *, 'ENTER THE Z COORDINATE IN WGS 84 OF THE ',
     &            'REFERENCE MARK'
         READ *, RZ
      ELSE
         STOP
      END IF
```

```
      IF (RESPN1 .EQ. 'Y') THEN
         PRINT *,'DO YOU WANT TO CHANGE THE CHECH MARK ("Y" OR ',
     &            '"N")'
         READ *, RESPN3
         IF (RESPN3 .EQ. 'Y') GO TO 30
         GO TO 40
      END IF
 20   PRINT *,'DO YOU WANT TO CHANGE THE REFERENCE MARKS ("Y" ',
     &         'OR  "N")'
      READ *, RESPN1
      IF (RESPN1 .EQ. 'Y') GO TO 10
 30   PRINT *,'ENTER THE X COORDINATE IN WGS 84 OF THE CHECK.',
     &         'MARK'
      READ *, CX
      PRINT *,'ENTER THE Y COORDINATE IN WGS 84 OF THE CHECK MARK'
      READ *, CY
      PRINT *,'ENTER THE Z COORDINATE IN WGS 84 OF THE CHECK MARK'
      READ *, CZ
      PRINT *,'ENTER THE ELLIPSOID DIFFERENCE, DH'
      READ *, DH
      PRINT *,'ENTER THE GLOBAL GEOID HEIGHT OF THE CHECK MARK',
     &          ',  GN'
      READ *, GN
      PRINT *,'ENTER THE LEVELED OTHOMETRIC HEIGHT OF THE CHECK '
     &         , 'MARK'
      READ *, LH
      DX = CX - RX
      DY = CY - RY
      DZ = CZ- RZ
      IF (IPA .EQ. IPA1 .AND. RESPN1 .EQ. 'N' .AND. IPA .EQ. 5) GO TO 50
      IF (IPA .EQ. IPA1 .AND. RESPN1 .EQ. 'N' .AND. IPA .EQ. 6) GO TO 60
 40   IF (IPA .EQ. 5) THEN
         PRINT *,'ENTER THE ELLIPSOID HEIGHT H0'
         READ *, H0
         PRINT *,'ENTER THE COEFFICIENT A'
         READ *, A
         PRINT *,'ENTER THE COEFFICIENT B'
         READ *, B
         PRINT *,'ENTER THE COEFFICIENT C'
         READ *, C
         PRINT *,'ENTER THE COEFFICIENT D'
         READ *, D
C
C THE 5-PARAMETER GEOID MODEL
C
 50      H = -GN + DH + H0 + A*DY + B*DX**2 + C*DY**2 + D*DX*DY
      ELSE
         PRINT *,'ENTER THE ELLIPSOID HEIGHT H0'
         READ *, H0
         PRINT *,'ENTER THE COEFFICIENT A''
         READ *, A1
         PRINT *,'ENTER THE COEFFICIENT B''
         READ *, B1
```

```
         PRINT *,'ENTER THE COEFFICIENT C'"
         READ *, C1
         PRINT *,'ENTER THE COEFFICIENT D'"
         READ *, D1
         PRINT *,'ENTER THE COEFFICIENT E'"
         READ *, E1
C
C THE 6-PARAMETER GEOID MODEL
C
 60      H = -GN + DH + H0 + A1*DX + B1*DZ + C1*DX**2 + D1*DY**2
     &      + E1*DX*DY
      END IF
C
C COMPUTE THE DIFFERENCE AND PRINT THE RESULT
C
      DIF = ( H - LH) * 100.
      WRITE(IW, 1) IPA
      WRITE(IW, 2) LH, H, DIF
 1    FORMAT (5X, I1, ' PARAMETERS')
 2    FORMAT (/5X,'H-LEVELING   =', F10.3, ' M'
     &         /5X,'H-GPS        =', F10.3, ' M'
     &         /5X,'DIFFERENCE   =', F10.3, ' CM'/)
      IPA1 = IPA
      H = 0.
      DIF = 0.
      PRINT *, 'MORE COMPUTATION ("Y" OR "N")'
      READ *, RESPN2
      IF (RESPN2 .EQ. 'Y' ) GO TO 20
      STOP
      END
```

APPENDIX D. FORTRAN PROGRAM DISTCO

```
***********************************************************************
*    WRITTEN BY MA, WEI_MING   05/11/88                               *
*    THIS PROGRAM COMPUTES THE DISTANCE BETWEEN THE BENCH             *
*    MARKS THEN CALLS THE SUBROUTINE C TO CALCULATE THE               *
*    COVARIANCE BETWEEN THE BENCH MARKS.                              *
*    THE VARIABLES USED ARE:                                          *
*       X,Y,Z   :   THE COORDINATES OF X,Y,Z, IN WGS 84 (m)           *
*        DIS    :   THE DISTANCES BETWEEN THE MARKS (m)               *
*        CQ     :   THE COVARIANCES BETWEEN THE BENCH MARKS           *
*                                                                     *
***********************************************************************
      REAL*8      X(10), Y(10), Z(10)
      REAL*8      DIS(10,10), CQ(10,10)
      DATA   DIS/100*0./, CQ/100*0./
      IW = 9
      N = 7
C
C READ X, Y, Z FROM THE DATA FILE
C
      CALL EXCMS('FILEDEF 8 DISK MBXYZ DATA A1')
      DO 100 I = 1, N
         READ(8,*) X(I), Y(I), Z(I))
  100 CONTINUE
C
C COMPUTE THE DISTANCE AND PRINT THE RESULTS
C
      DO 300 I = 1, N
         WRITE(IW,1) I
 1       FORMAT(/1X,'FROM BENCH MARK #', I2,'   TO', 'DISTANCE (m)'/
     &             1X, 53('-'))
         DO 200 J = 1, N
            IF (J .EQ. I)  GO TO 10
            DIS(I,J) = SQRT((X(J) - X(I)) ** 2 + (Y(J) - Y(I)) ** 2
     &                 +(Z(J)-Z(I))**2)
 10         WRITE(IW,2) J, DIS(I,J)
 2          FORMAT(11X, I2, 14X, G20.8)
 200     CONTINUE
 300  CONTINUE
C
C CALL SUBROUTINE  C TO CALCULATE THE CQ AND PRINT THE RESULTS
C
      WRITE(IW,3)
 3    FORMAT(/3X, 'CQ = ')
      CALL C(DIS,N,CQ)
      DO 400 J = 1, N
         WRITE(IW,4)  (CQ(I,J), I = 1, N)
 4       FORMAT(3X, 7(G16.7, 2X)
 400  CONTINUE
```

```
      STOP
      END
      SUBROUNTINE C(DIS,N,CQ)
*************************************************************************
*                                                                       *
*    WRITTEN BY MA, WEI_MING    05/11/88                                *
*    THIS SUBROUTINE COMPUTES THE COVARIANCES OF THE RANDOM *
*    ERROR DISTRIBUTION FUNCTION:                                       *
*    C(R) = A + B * SIN(D*DIS)                                          *
*    THE VARIABLES USED ARE:                                            *
*         A     :   STANDARD DEVIATION OF THE OBSERVATIONS              *
*         B     :   CONSTANT                                            *
*         C     :   CONSTANT (RADIANS/METERS)                           *
*        DIS    :   DISTANCE (METERS)                                   *
*                                                                       *
*************************************************************************
      REAL*8 DIS(10,10), CQ(10,10), A, B, D
      A = 0.137153
      B = 0.147951
      D = 2.85958
      DO 200 J = 1, N
         DO 100 I = 1, N
            CQ(I,J) = A - B * SIN(D * DIS(I,J) / 33300.)
 100     CONTINUE
 200  CONTINUE
      RETURN
      END
```

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